Strategic Design for Delivery with Linked Transportation Assets: Trucks and Drones

Final Report
January 2018

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16. Abstract
Home delivery by drones as an alternative or complement to traditional delivery by trucks is attracting considerable attention from major retailers and services, as well as startups. While drone delivery may offer considerable economic savings, the fundamental issues of how best to deploy drones for home delivery are not well understood. Our research provides a strategic analysis for the design of hybrid truck-drone delivery systems using continuous approximation modeling techniques to derive general insights. We formulated and optimized models of hybrid truck-drone delivery, where truck-based drones make deliveries simultaneously with trucks, and compared their performance to truck-only delivery. Our results suggest that truck-drone delivery can be very advantageous economically in many settings, especially with multiple drones per truck, but the benefits depend strongly on the relative operating costs and marginal stop costs.

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Principal Investigators

James F. Campbell, Professor and Chair
Logistics & Operations Management
University of Missouri – St. Louis

Donald C. Sweeney II, Associate Director and Teaching Professor
Center for Transportation Studies and Logistics & Operations Management
University of Missouri – St. Louis

Research Assistants
Juan Zhang and Deng Pan

Authors
James F. Campbell, Donald C. Sweeney II, Juan Zhang, and Deng Pan

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1. INTRODUCTION

Home delivery by drones is being promoted and researched by a growing number of firms as a possible alternative or complement to traditional delivery by trucks. Though many of the details about drone delivery remain uncertain, recent reports suggest that drones might make small package deliveries for only $1 (Soper 2015, Keeney 2015). Further interest stems from reports that 86% of Amazon.com's orders are less than 5 lbs (ideal for drone delivery) (Rose 2013) and that 82% of customers are willing to pay for drone delivery (Snow 2014). Thus, drones provide new opportunities to improve home delivery services, and therefore research is required on how best to deploy drones for home delivery.

Proposals for drone delivery vary widely, with drones being used independently or in conjunction with delivery by trucks. In this report, we address hybrid truck-drone delivery, where drones are autonomous vehicles (i.e., unmanned) that make deliveries on their own, with each drone departing from and returning to a truck. While our main focus is on aerial drones that generally have a small payload capacity (e.g., 5 lbs), our modeling can equally apply to ground-based drones that can carry considerably larger loads (e.g., Pettitt 2015, Pymnts 2015).

While widespread use of delivery drones requires overcoming a variety of economic, regulatory, and technological obstacles (Lee et al. 2016), there is a need for strategic analyses of truck-drone delivery systems to help identify promising delivery system designs. Note that our research is oriented toward home delivery systems, not military or surveillance applications, though there are some obvious similarities. There are also many related healthcare applications (e.g., Markoff 2016, Thiels et al. 2015, Francisco 2016), but our focus is on delivery of small parcels to homes and businesses (i.e., "last-mile delivery" as currently done by UPS, FedEx, US Postal Service, etc.). Our research is based on operations such as the systems used by HorseFly (Workhorse 2017), the partnership between Matternet and Mercedes-Benz (Sloat and Kopplin 2016, Matternet 2017), and UPS (UPS 2017). In this report, our focus is on minimizing expected costs for deliveries across a region, such as by replacing long (e.g., eight-hour) truck-only delivery routes by hybrid truck-drone routes.

The fundamental research questions we address are as follows:

- How can hybrid truck-drone routes best be used to serve a region?
- How do hybrid truck-drone routes compare with truck-only delivery?

Because many different types of delivery drones continue to be proposed, our flexible modeling framework allows us to analyze a range of drone types and delivery strategies. As an illustration, Figure 1 (left) shows a hybrid truck-drone delivery route, where the truck and drone alternate deliveries while traveling through the service region.
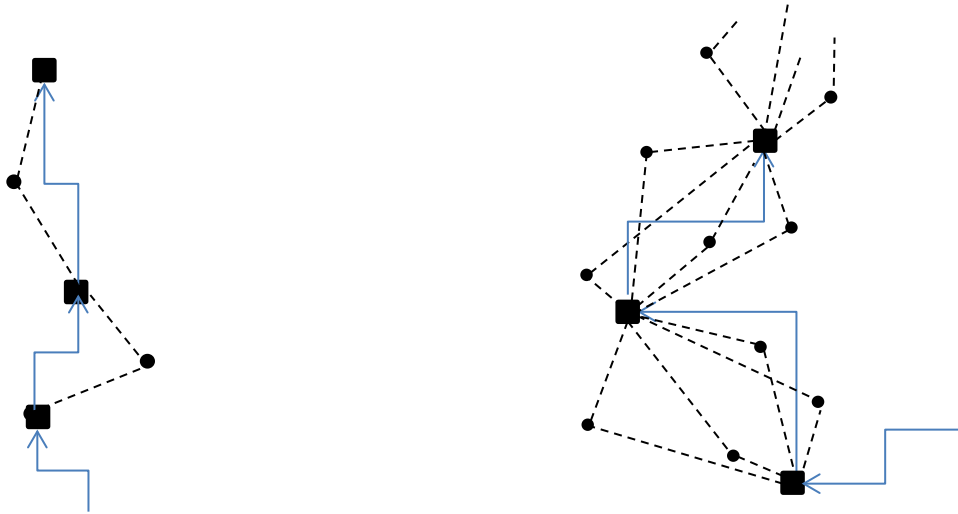
Figure 1. Hybrid truck-drone delivery with alternating deliveries (left) and multiple drone deliveries per truck delivery (right)

The truck route is shown with rectilinear travel (L1 metric) between the truck deliveries, indicated by squares, while the drone uses straight-line travel (L2 metric) to deliver to the customers, indicated by circles. Figure 1 (right) illustrates hybrid truck-drone delivery, where a truck route has three delivery stops, from each of which are four single-stop drone routes.

For strategic analyses, we formulated continuous approximation (CA) models of truck-drone delivery, where trucks transport drones from a depot to the vicinity of customers, and then both drones and trucks make deliveries simultaneously, with the drones launched from and returning to the truck (to pick up more items for delivery). We also modeled truck-only delivery on multi-stop tours for comparison. In this work, we considered only one-stop drone routes, where each drone carries a single item for delivery. We employed reasonable parameter values and operating characteristics in the models to evaluate a range of delivery environments (e.g., from very rural to suburban regions) and highlighted key areas of importance for beneficial drone operations. Our focus was on logistical and operational aspects, and not on the (important) legal, regulatory, and technological issues, including customer acceptance (Lee et al. 2016, Lotz 2105).

The remainder of the report is organized as follows. Chapter 2 presents a review of relevant literature on truck-drone delivery modeling. Chapter 3 provides the basic continuous approximation model formulations. Chapter 4 describes the development of reasonable values for drone operating costs. Chapter 5 provides the results of several analyses for a range of settings, and Chapter 6 offers conclusions.

2. LITERATURE REVIEW

Researchers have begun to analyze truck-drone delivery systems by extending traditional discrete Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) models and solution methods. Perhaps the earliest directly relevant research is Lin (2008, 2011), which considers "transportable delivery resources" carried on trucks. In these studies, foot couriers were the transportable resource, and both the couriers and trucks could make deliveries, with trucks travelling much faster (20 km/h versus 5 km/h for foot couriers). These studies contrast with the recent research on truck-drone delivery, which has largely been directed at aerial drones that are typically much faster than trucks.

Murray and Chu (2015) analyze a TSP variant with a single truck and single drone, called the flying sidekick TSP (FSTSP), where the drone and truck start and end at a depot, the drone travels on the truck, and the drone can make deliveries on its own. Each drone delivery involves a single stop, with the drone departing from and returning to the truck at sequential truck delivery stops. The objective is to minimize the total time for both the truck and drone to return to the depot, which includes travel time, launch and recovery time for the drone, and waiting time for the drone or truck at the stop where recovery occurs. Murray and Chu (2015) also consider a parallel version of the problem where the truck and drone serve different sets of customers concurrently, with the drone traveling back and forth between the depot and customers while a truck visits other customers. For both formulations, Murray and Chu (2015) provide simple construction and improvement heuristics for solution.

Ponza (2016) considers the same problem and presents an improved mathematical formulation and a simulated annealing solution approach. Ha et al. (2015) consider a very similar problem, though with a different name (TSP-D). In this problem, once the drone is launched from either the depot or the truck (at a customer stop), it either returns to the truck at the next stop of the truck or it returns to the depot. Ha et al. (2015) optimize an objective that may be the number of drone trips, travel time, or travel distance, and solutions are presented for cluster first–route second and route first–cluster second heuristics. Ha et al. (2016) consider a similar setting, but with the objective of minimizing the sum of the truck and drone costs, and present two heuristic solution methods based on greedy randomized adaptive search procedure (GRASP) and modification of an optimal TSP tour.

Agatz et al. (2015) consider a variant of the truck-drone delivery problem where the drone may launch and recover at the same truck stop, and the authors provide solutions from a route first–cluster second heuristic. Ferrandez et al. (2016) analyze a different model where the truck follows a TSP tour, and from each truck stop one or more drones can perform multiple single-stop deliveries. Thus, each truck stop is essentially a hub for multiple drone deliveries. The solution approach uses K-means clustering to find the truck stops and a genetic algorithm for finding the truck route. Dorling et al. (2016) use a model for drone energy consumption to develop cost and time minimization models for multi-stop drone deliveries from a depot. A different perspective is explored by Wang et al. (2016), which provides worst-case analyses for several truck-drone delivery settings, including the use of multiple truck-drone pairs. On a

related theme, Hong et al. (2015) present a model for locating drone recharging stations and routing drones among these stations.

In contrast to the discrete TSP-based optimization models described above, our research developed strategic models for the design of truck-drone delivery systems using continuous approximation (CA) modeling techniques, where the demand for deliveries is treated as a continuous spatial density over a service region. For background and applications of CA models, see Campbell (1993), Langevin et al. (1996), and Daganzo et al. (2012). Carlsson and Song (2017) use CA models for an asymptotic analysis of a truck-drone delivery system where all deliveries are made by drones that are launched and recovered at trucks as the trucks make a tour through the service region. The objective of the analysis was to minimize the time of the last delivery, and costs were not modeled.

In our research, we developed mathematical models for truck-drone delivery using CA techniques by extending the swath model pioneered by Daganzo (1984a). We formulated mathematical expressions for the expected travel cost (and time) for the trucks and drones based on the strategy employed for deploying the trucks and drones, the fundamental parameters of the problem setting (e.g., spatial density of customers [demand] and the fraction of demand served by drones), and the operating characteristics of the trucks and drones (e.g., operating costs, stop costs, and speeds). The models allow different travel metrics because truck travel is via roads while aerial drones may travel (more or less) in straight lines. In contrast to Carlsson and Song (2017), our models allow comparison of expected costs for truck-drone and truck-only delivery and an evaluation of the sensitivity of results to operating costs and the delivery density.

3. CONTINUOUS APPROXIMATION MODELS

This chapter provides a derivation of the continuous approximation truck-drone delivery model. The fundamental idea is to extend the "strip" modeling strategy for approximating the distance of vehicle routes from Daganzo (1984a) to allow the truck to launch and recover drones at truck stops (customers). For modeling purposes, let the service region be a compact region of area A where delivery stops are randomly and independently distributed in the region with density δ.

3.1 Truck-Only Delivery

Consider a truck traveling using the L1 metric along the length of a swath of width w, like that shown in Figure 2, and visiting all customers in (vertical) order along the swath.

w

Figure 2. Four truck stops along a swath of width w

The expected horizontal distance between adjacent stops is $w/3$, and the expected vertical distance between adjacent stops is $1/w\delta$. If a truck is to visit all stops, combining the expected horizontal and vertical truck travel distance between two adjacent stops gives an expected total truck travel (L1) distance per stop of

$$\frac{w}{3} + \frac{1}{\delta w}. \tag{1}$$

The optimal swath width to minimize the expected truck-only travel distance is $w_{to}^* = \sqrt{\frac{3}{\delta}}$, and

the minimal expected truck travel distance would be $2\sqrt{\frac{1}{3\delta}}$.

The model above is appropriate for a truck that leaves the depot and travels along the swath making deliveries, as in Figure 3 (left), where the entire service area can be served with five routes from a centrally located depot, with each route serving a wedge-shaped zone.

Figure 3. No linehaul travel when few routes cover the service region (left) and linehaul travel when many routes are in the service region (right)

However, when each route serves only a small portion of the service region, then the total expected travel distance should include a "linehaul" portion from the depot to the zone where the deliveries on a route begin. Figure 3 (right) shows the start of two of the many routes in the service region when each route serves only a small part of the region and linehaul travel is needed from the depot to the zone. To formulate the e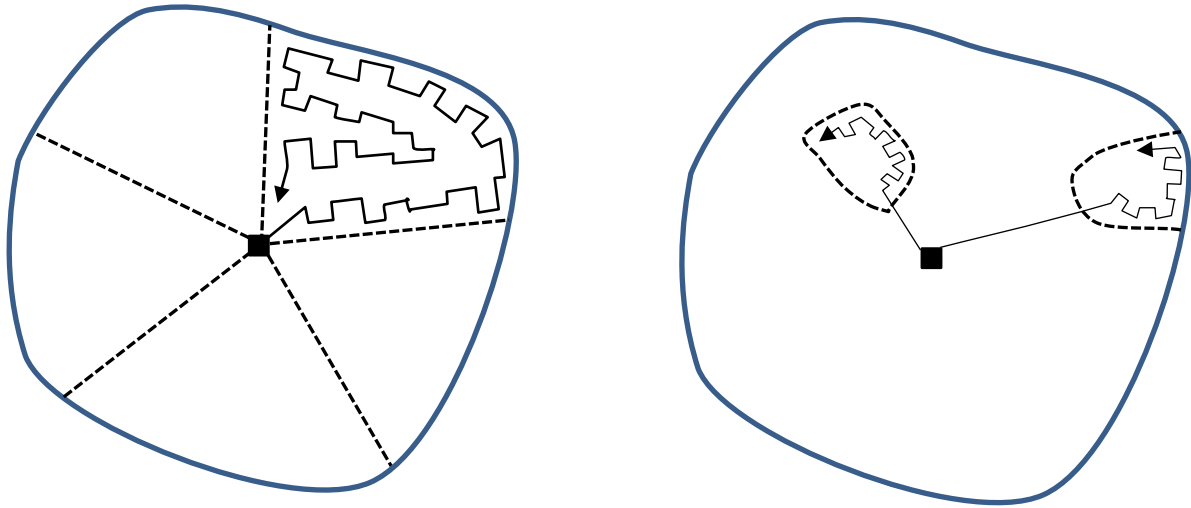xpected linehaul travel, suppose the truck has a delivery capacity of serving m_t stops. Let the depot be centrally located in a compact service region of area A, and let ρ be the expected distance from the depot to a delivery stop in a zone, where zones are reasonably compact. For a circular service region, $\rho = \frac{2}{3}\sqrt{A/\pi}$, and similar values hold for other compact shapes of the service region (see Eilon et al. 1971).

However, the distance (and cost) can be reduced by using linehaul travel only to the edge of the zone and defining a zone shape that is elongated in the direction of the depot (see Daganzo 1984b). Suppose we use rectangular zones of length l and a width equal to twice the swath width w. Figure 4 shows an example where the linehaul travel to the edge of the zone is shown with a dashed line.
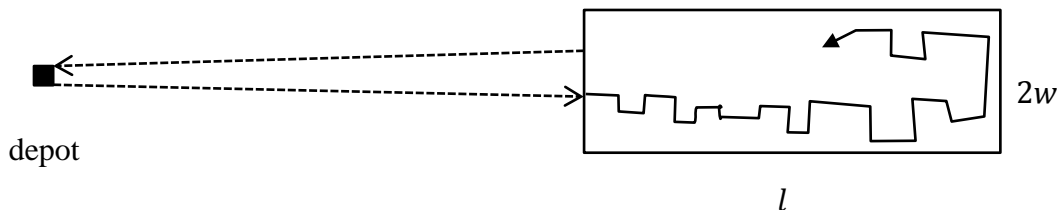

Figure 4. An elongated zone reduces linehaul travel

Because the zone has a width of $2w$, the delivery truck travels down one side and back the other side. The expected linehaul distance to the edge of the zone can be written

$$\rho - \frac{l}{2} = \rho - \frac{m_t}{4w\delta}, \tag{2}$$

using $l \times 2w = \frac{m_t}{\delta}$, because there are m_t stops in the zone. The expected round trip linehaul distance per delivery is then

$$\frac{2}{m_t}\left(\rho - \frac{l}{2}\right) = \frac{2\rho}{m_t} - \frac{1}{2w\delta}, \tag{3}$$

which shows that the savings per delivery from not traveling to the center of the zone are effectively one-half of the expected horizontal travel distance $\left(\frac{1}{2w\delta}\right)$; see equation (1).

Adding the round trip linehaul transportation to equation (1) gives a total truck travel distance of $\frac{2\rho}{m_{to}} + \frac{w}{3} + \frac{1}{2\delta w}$, for which the optimal swath width is $w_{to}^* = \sqrt{\frac{3}{2\delta}}$. Using this optimal swath width gives a total expected cost per stop for truck-only delivery:

$$E_{to} = c_t \left\{\frac{2\rho}{m_{to}} + \frac{\sqrt{2}}{\sqrt{3\delta}}\right\} + s_t. \tag{4}$$

In the remainder of this report, we will focus on the case with linehaul transportation, which is used when many routes are used to serve a region.

3.2 Hybrid Truck-Drone Delivery

Now suppose that drones can also visit customers to make deliveries in place of the truck, and the drones are launched and recovered by the truck. Figure 5 (left) shows the truck and drone alternating deliveries over two "cycles" (moving from bottom to top), where the drone travel is indicated by dashed lines and the truck travel, indicated by solid lines, has arrows showing direction of travel.
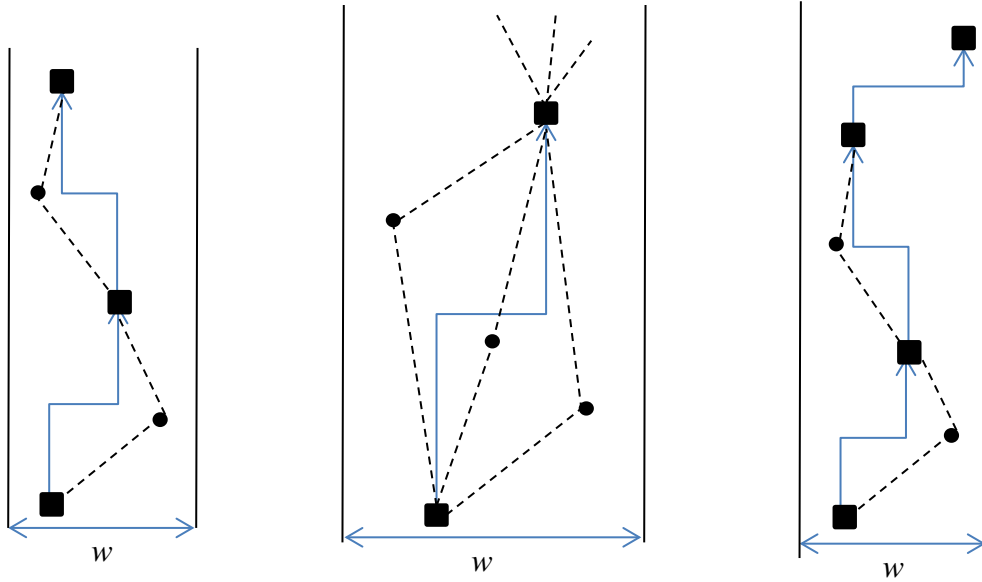
Figure 5. Truck-drone delivery for *n* = 1 (left), *n* = 3 (center), and *n* = 2/3 (right)

If drone travel is via Euclidean distance (the L2 metric) and the drone launches and is recovered at the truck delivery customers adjacent to the drone delivery customers (in the vertical direction), then the expected drone travel distance per point is

$$\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2}. \tag{5}$$

To generalize to situations where the drone and truck do not simply alternate deliveries, we modeled the truck-drone travel as a connected series of cycles, with *n* representing the ratio of the number of drone deliveries in one cycle to the number of truck deliveries in one cycle. Then we considered two cases: $n > 1$, where there are multiple drone stops per truck stop, and $0 < n \leq 1$, where the drone makes the same ($n = 1$) or fewer stops than the truck per cycle.

3.3 *n* > 1: Multiple Drone Deliveries per Truck Delivery

For $n \geq 1$, there are more drone deliveries than truck deliveries in a cycle (or the same number of truck and drone deliveries for $n > 1$). For $n > 1$, there is a single drone trip departure and return at each truck delivery, as in Figure 5 (left); however, there are several operational possibilities associated with $n > 1$, including having multiple drones per truck each making a single delivery per cycle, having a single drone per truck making multiple deliveries per cycle, or some combination of these. Our interest here is exploring the benefits of multiple drones per truck, so we considered a situation where *n* drones launch from the truck at each truck delivery stop; the drones make one delivery each, visit the next *n* stops along the strip; and then recover all the drones at the delivery truck's *n+1st* stop. Thus, there is a single truck delivery in each cycle, and *n* is the number of drones per truck as well as the ratio of drone deliveries to truck deliveries per

cycle. Figure 5 (center) illustrates a case where $n = 3$. Other options with $n > 1$, such as having a single drone launch and return to the same truck stop repeatedly, are an area of ongoing research.

For a truck-drone delivery with $n \geq 1$, the expected horizontal distance between adjacent truck deliveries is $w/3$ and the expected vertical truck travel distance between adjacent truck deliveries is $(n+1)/w\delta$. The linehaul travel for truck-drone delivery to the nearest edge of a rectangular zone can be formulated as before for truck-only travel as

$$\frac{2}{m_{td}}\left(\rho - \frac{l}{2}\right) = \frac{2\rho}{m_{td}} - \frac{1}{2w\delta},$$

(6)

where m_{td} is the number of deliveries for the truck-drone route (in a zone of width $2w$). Combining linehaul travel with the expected horizontal and vertical truck travel distance between two adjacent deliveries gives an expected truck travel distance per delivery of $\left(\frac{2\rho}{m_{td}} + \frac{w}{3}\frac{1}{n+1} + \frac{1}{2\delta w}\right)$. Considering the truck travel alone, the optimal swath width is given by

$$w_t^* = \sqrt{\frac{n+1}{2}}\sqrt{\frac{3}{\delta}}.$$

(7)

With drone travel via Euclidean distance (the L2 metric) and all n drones launched from a single truck stop and recovered at the $n+1st$ truck stop, the total expected drone travel distance for the n delivery stops in a cycle is

$$\sum_{i=1}^{n}\left(\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{i}{\delta w}\right)^2} + \sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1-i}{\delta w}\right)^2}\right).$$

(8)

In equation (8), we can approximate the distance for a drone delivery using the distance for the average drone delivery, which is halfway between the truck stops, or a vertical distance of $\frac{n+1}{2}\frac{1}{\delta w}$ from each truck stop. This is a very good approximation (errors $< 2\%$) unless the drone delivery is very close to the truck delivery. Replacing each square root in the summation by distance $\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2}$ yields an expected approximate drone distance per delivery of

$$\frac{2n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2}.$$

(9)

For $n = 1$, this expression is exact because the expected drone distance per delivery is $\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2}$. The optimal swath width that minimizes the drone travel distance (equation (9)) is then

$$w_d^* = \sqrt{\frac{n+1}{2}}\sqrt{\frac{3}{\delta}},$$

(10)

which is the same as for the truck travel alone.

Let c_d be the drone variable cost per unit distance and s_d be the marginal cost for drone delivery (including launch and recovery costs) relative to truck delivery. Thus, the delivery stop cost for a drone is given by $s_t + s_d$, where a negative (positive) value for s_d is the per delivery savings (excess costs) for making a delivery with a drone. (If truck and drone delivery costs are equal, then $s_d = 0$.) The expected total cost per delivery for hybrid truck-drone delivery with $n \geq 1$, including truck travel, drone travel, and the stop cost, can then be approximated as

$$E_{td}(n,w) = c_t\left[\frac{1}{n+1}\frac{w}{3} + \frac{1}{\delta w}\right] + c_d\frac{2n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2} + c_t\frac{2\rho}{m_{td}} + \frac{n}{n+1}s_d + s_t.$$

(11)

For the optimal swath width $w_{td}^* = \sqrt{\frac{n+1}{2}}\sqrt{\frac{3}{\delta}}$, the expected cost is

$$E_{td}(n,w_{td}^*) = c_t\left[\frac{1}{n+1}\frac{w}{3} + \frac{1}{\delta w}\right] + c_d\frac{2n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2} + c_t\frac{2\rho}{m_{td}} + \frac{n}{n+1}s_d + s_t$$

(12)

$$= c_t\sqrt{\frac{2}{3}}\frac{1+\sqrt{2}n\frac{c_d}{c_t}}{\sqrt{n+1}}\sqrt{\frac{1}{\delta}} + c_t\frac{2\rho}{m_{td}} + \frac{n}{n+1}s_d + s_t.$$

We can write

$$E_{td}(n,w_{td}^*) = c_t k_{td,n\geq1}\sqrt{\frac{1}{\delta}} + c_t\frac{2\rho}{m_{td}} + \frac{n}{n+1}s_d + s_t,$$

(13)

where the first term includes the parameter $k_{td,n\geq1} = \sqrt{\frac{2}{3}}\frac{1+\sqrt{2}n\frac{c_d}{c_t}}{\sqrt{n+1}}$ that depends only on the number of drones (n) and the cost ratio $\frac{c_d}{c_t}$; the second term depends on the expected distance from the depot to a delivery in the zone and the number of delivery stops; and the last two terms are the stop cost.

3.4 0 < n ≤ 1: One Drone per Truck

For $n \leq 1$, we modeled the drone launching and returning to the truck at different customers because the drone makes the same (for $n = 1$) or fewer stops than the truck. Thus, we modeled travel along the swath such that the drone delivery stop is the first customer after the truck stop where the drone is launched, and the drone is recovered at the first customer (truck stop) immediately after the drone stop. (This is the same setup used in much of the discrete modeling

for truck-drone delivery noted earlier.) As an example, Figure 5 (right) shows $n = \frac{2}{3}$, where the drone stays on the truck between the last two truck stops. The fraction of stops served by the drone is denoted f, and $f = \frac{n}{n+1}$ ($f = 2/5$ in Figure 5(right)).

For $n \leq 1$, the expected truck horizontal travel distance per point in one cycle is $\frac{w}{3}\left(1 - \frac{n}{n+1}\right) = \frac{w}{3}\frac{1}{n+1}$, and the expected truck vertical travel distance per point in one cycle is $\frac{1}{\delta w}$ (the same as before because the trucks must pass all stops in the vertical direction). Thus, the expected total truck travel distance per point is

$$\frac{w}{3}\frac{1}{n+1} + \frac{1}{\delta w}. \tag{14}$$

The optimal swath width to minimize the expected truck travel distance or time (assuming a constant truck speed) is

$$w_t{}^* = \sqrt{n+1}\sqrt{\frac{3}{\delta}}. \tag{15}$$

Because the fraction of stops made by the drone is $\frac{n}{n+1}$, the expected drone travel distance per

point is

$$2\frac{n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2}, \tag{16}$$

where the square root term provides the expected Euclidean distance for one leg of the drone trip. The optimal swath width to minimize the expected drone travel distance (or travel time assuming a constant drone speed) is the same as for using a truck alone and is given by

$$w_d{}^* = \sqrt{\frac{3}{\delta}}. \tag{17}$$

We combine the truck and drone travel to model the expected total cost of the truck-drone route with $n \leq 1$. Based on the expected distances (14) and (16), the total expected cost per delivery when $n \leq 1$ is

$$c_t\left[\frac{w}{3}\frac{1}{n+1} + \frac{1}{\delta w}\right] + 2c_d\frac{n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2} + \frac{n}{n+1}s_d + s_t. \tag{18}$$

In terms of the fraction of stops made by drones $f = \frac{n}{n+1}$, the total expected cost per stop is

$$c_t \left[\frac{w}{3}(1-f) + \frac{1}{\delta w} \right] + 2c_d f \sqrt{\left(\frac{w}{3} \right)^2 + \left(\frac{1}{\delta w} \right)^2} + fs_d + s_t, \tag{19}$$

which is linear in *f* for a fixed swath width. Thus, the optimal fraction of drone deliveries to minimize costs is either $f = n = 0$, i.e., using truck-only delivery and no drones, or $f = 0.5$ (when $n = 1$), i.e., using drones for half the deliveries (as much as possible). This latter case (using drones for half the deliveries) is preferred when the ratio of drone to truck operating costs per unit distance is small enough:

$$\frac{c_d}{c_t} < \frac{\frac{w}{3} - \frac{s_d}{c_t}}{2\sqrt{\left(\frac{w}{3} \right)^2 + \left(\frac{1}{\delta w} \right)^2}}. \tag{20}$$

The numerator of the expression on the right side of equation (20) is the expected horizontal truck travel distance between two stops minus the ratio of the marginal drone stop cost to the truck cost per mile. This equation shows how the desirability of using drones depends on their operating cost relative to trucks, the density of deliveries (δ), the magnitude of the marginal drone stop cost relative to the truck operating cost $\left(\frac{s_d}{c_t} \right)$, and the design parameter *w* (width of the swath).

The minimum expected total cost per stop is realized with the optimal swath width that minimizes expression (18), but this does not lend itself to a simple closed-form solution. However, because both the truck and drone components of the expected travel cost are convex, the optimal swath width *w** will be between the values for using drones alone or trucks alone (equations (15) and (17)). Extensive numerical experimentation has shown that the nearly optimal swath width is given by the expression

$$w_{td,n\leq 1}^* \cong \frac{\left(\sqrt{n+1} + 2\frac{c_d}{c_t} \right)}{1 + 2\frac{c_d}{c_t}} \sqrt{\frac{3}{\delta}} = k_1 \sqrt{\frac{3}{\delta}}, \tag{21}$$

where $k_1 = \frac{\left(\sqrt{n+1} + 2\frac{c_d}{c_t} \right)}{1 + 2\frac{c_d}{c_t}}$ and we suppress the dependence of k_1 on *n, c_t,* and c_d. Using k_1, the expected cost per delivery (expression 18) can be written

$$\frac{1}{\sqrt{3\delta}} \left\{ c_t \left[\frac{k_1}{n+1} + \frac{1}{k_1} \right] + 2c_d \frac{n}{n+1} \sqrt{k_1^2 + \frac{1}{k_1^2}} \right\} + \frac{n}{n+1} s_d + s_t, \tag{22}$$

where the term in brackets depends only on *n, c_t, c_d,* and s_d but not on the density of stops δ.

For truck-drone travel with $n \leq 1$, adding the round trip linehaul transportation to equation (18) gives a total cost per stop of

$$= c_t \frac{2\rho}{m_{td}} + c_t \left[\frac{w}{3} \frac{1}{n+1} + \frac{1}{2\delta w} \right] + 2c_d \frac{n}{n+1} \sqrt{\left(\frac{w}{3} \right)^2 + \left(\frac{1}{\delta w} \right)^2} + \frac{n}{n+1} s_d + s_t. \tag{23}$$

The optimal swath width w^* for this cost expression can be written

$$w^* \cong \frac{\left(\sqrt{n+1} + 2\frac{c_d}{c_t} \right)}{\sqrt{2} + 2\frac{c_d}{c_t}} \sqrt{\frac{3}{\delta}} = k_{1l} \sqrt{\frac{3}{\delta}}, \tag{24}$$

where k_{1l} is the parameter for determining the optimal swath width when linehaul distance is included and k_1 is used when linehaul distance is not included.

3.5 Truck Travel Time and Number of Stops per Route

The expected travel time of a route can be calculated using the expected distance relations derived earlier. Let c_t' be the reciprocal of the average truck speed while delivering, c_{tl}' be the reciprocal of the average truck linehaul speed, c_d' be the reciprocal of the average drone speed, s_t' be the average stop time per truck delivery at a customer, and s_d' be the marginal stop time per drone delivery relative to the truck stop time per delivery. Thus, if $s_t' = 1$ minute and the average drone stop time for a drone delivery is 0.6 minutes, then $s_d' = -0.4$ minutes. With m_{td} total truck and drone stops per route, the number of drone delivery stops per route is $\frac{m_{td} \times n}{n+1}$ and the number of truck delivery stops per route is $\frac{m_{td}}{n+1}$. The expected travel time for a truck on a truck-drone route with m total deliveries is

$$m_{td} \times \left[c_t' \left(\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right) + \frac{1}{n+1} s_t' - c_{tl}' \frac{1}{2\delta w} \right] + 2\rho c_{tl}' \ .$$

This expected time assumes that the truck does not need to wait for the drone (as in the illustrations that follow). With an available time for the route of T hours, the number of delivery stops for a truck-drone route is given by

$$m_{td} = \frac{T - 2\rho c_{tl}'}{c_t' \left[\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right] - c_{tl}' \frac{1}{2\delta w} + \frac{1}{n+1} s_t'} \quad \text{or} \tag{25}$$

$$m_{td} = \frac{T}{c_t' \left[\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right] + \frac{1}{n+1} s_t'}, \tag{26}$$

depending on whether linehaul travel is needed. The number of delivery stops for a truck-only route can be derived similarly as

$$m_{to} = \frac{T - 2\rho c_{tl}'}{c_t' \left[\frac{w}{3} + \frac{1}{\delta w} \right] - c_{tl}' \frac{1}{2\delta w} + s_t'} \quad \text{or} \tag{27}$$

13

$$m_{to} = \frac{T}{c'_t \left[\frac{2}{\sqrt{3\delta}}\right] + s'_t} .$$

(28)

4. DRONE OPERATIONAL AND COST DATA

Evaluation of potential benefits from truck-drone delivery systems using the expected cost formulae requires reasonable values of the relevant parameters, especially drone costs. Unfortunately, we cannot collect such data in the field because drone-based delivery systems are not currently in operation. Further, the firms experimenting with drone delivery have not published operational and performance data from an actual implementation, and any future deployment of drone delivery systems may well incur costs different than those of today's drones. To further complicate matters, there is a wide range of data from academic, practitioner, and industry sources (in part due to the various types of drones envisioned).

One of the best attempts to analyze drone system costs and operating parameters at a strategic level is by ARK Invest (Keeney 2015). ARK Invest suggested a large-scale implementation (based on Amazon.com) could serve 400 million deliveries per year in the US, which includes packages under 5 lbs and within 10 mi of a depot. The company considered system costs for drone delivery from fixed depots where each drone makes an average of 30 deliveries per day in two eight-hour shifts and each drone operator is responsible for 10 to 12 drones. They estimated annual capital costs for infrastructure, drones, and batteries and annual operating costs for labor (drone operators), data bandwidth, maintenance, and electricity. This resulted in a total annual cost of $350 million and a suggested price of $1 per delivery (assuming a 15% discount rate). With an average seven-mile (one-way) drone delivery distance, the $1 cost per drone delivery equates to about $0.07/mi.

In this report, to explore a range of operating costs and the sensitivity of the model results to various operational parameter values, we consider $n = 1$ to 8 drones per truck, drone operating costs per mile from $c_d = \$0.01$ to $\$0.625$, and marginal drone stop costs from $s_d = -\$0.20$ to $\$0.10$. We also vary the density of deliveries widely to explore the benefits of truck-drone delivery in different settings. All models employ a baseline setting of a service region of $A = 1{,}250$ mi^2, a time limit of $T = 8$ hours per route, a truck operating cost of $c_t = \$1.25$ per mile, and a truck stop cost of $\$0.40$. We limit the truck capacity to a maximum of 500 delivery stops to reflect the size of delivery trucks currently in use. In all instances, we use a truck linehaul speed of 40 mi/h ($c_{tl}'=1/40$), a truck delivery speed of 20 mi/h ($c_t' = 1/20$), and a stop time of 1 minute per delivery for the truck and for the drone (i.e., $s_t' = 1$ minute and $s_d' = 0$).

5. RESULTS

For a particular problem instance (value of δ, n, c_d, and s_d), we calculate the swath width (w) and the number of truck and drone stops per route from the appropriate equation, limited to a maximum of 500 deliveries per route. The total expected costs per delivery are denoted E_{to} and $E_{td}(n)$ (to is for truck-only and td is for truck-drone). The percent savings per delivery from truck-drone delivery relative to truck-only delivery is

$$PSAV = \frac{E_{to} - E_{td}(n)}{E_{to}}.$$

Positive values of $PSAV$ indicate savings from truck-drone delivery relative to truck-only delivery, and negative values indicate that truck-only delivery has a lower expected cost. The following results highlight the potential benefits of truck-drone delivery and the impact of four key parameters of interest: the number of drones per truck n, the density of deliveries δ, the drone operating cost per mile c_d, and the drone marginal stop cost s_d.

5.1 Benefits from Multiple Drones per Truck

To illustrate the benefits from using multiple drones per truck, Figure 6 shows $PSAV$ with one to eight drones as a function of the drone operating cost c_d, where the marginal drone stop cost is zero and the delivery density is $\delta = 10$ per mi^2.
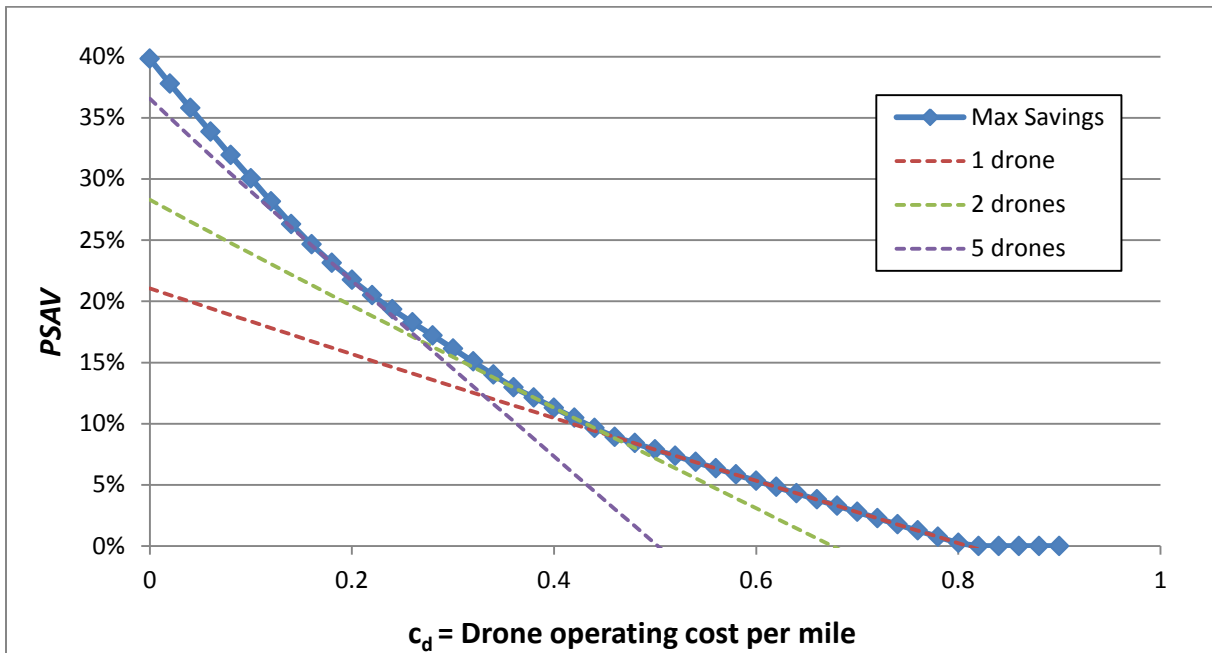

Figure 6. Percentage savings with one to eight drones with increasing drone operating cost for $\delta = 10$ and $s_d = 0$

The three dashed straight lines show the savings for a given number of drones per truck: $n = 1$, 2, or 5. The solid curve shows the maximum savings from allowing the number of drones per truck to vary with the drone operating cost (i.e., the upper envelope of the savings with one to eight drones per truck). With low drone operating costs ($c_d < 0.12$), eight drones per truck provides the largest savings (with a maximum of nearly 40%). Fewer drones per truck are used as c_d increases, and there is no savings for $c_d > 0.81$. With one drone per truck, the maximum savings (for a very small drone operating cost) is about 21%, and there are marginally decreasing additional savings from using more drones per truck. The maximum savings is 28.3% with two drones per truck, 32.6% with three drones per truck, 34.9% with four drones per truck, and 39.8% with eight drones per truck. The solid line in Figure 7 shows substantial benefits from multiple drones per truck when $c_d < 0.40$ (about one-third of the truck operating cost per mile), which seem to be very reasonable values for drone operating cost.

Figure 7. Swath width and number of drones per trucks with one to eight drones with increasing drone operating cost for $\delta = 10$ and $s_d = 0$

Figure 7 is a companion to Figure 6 that shows how the swath width and number of drones per truck decrease as the drone operating cost rises. This illustrates that as drones become more expensive to operate, fewer drones per truck should be used and the swath width should decrease to reduce travel in the direction perpendicular to the general direction of travel along the swath. Table 1 shows for selected values of c_d the percentage savings per delivery and the number of drones per truck that provide the largest savings when $s_d = 0$ (as in Figures 6 and 7) and $s_d = -0.2$ (drone delivery stops cost $0.20 less than truck delivery stops).

Table 1. Percentage savings ($PSAV$) and number of drones per truck (n) that produce the lowest cost per stop with increasing drone operating cost for $\delta = 10$

c_d	$s_d = 0$		$s_d = -0.2$	
	$PSAV$	n	$PSAV$	n
0.0	39.8%	8	56.7%	8
0.1	30.0%	8	47.0%	8
0.14	26.3%	7	43.2%	8
0.16	24.6%	6	41.3%	8
0.18	23.1%	5	39.5%	7
0.2	21.8%	4	37.8%	6
0.24	19.3%	3	34.7%	5
0.3	16.1%	3	30.7%	4
0.4	11.3%	2	25.1%	3
0.5	7.9%	1	19.9%	2
0.6	5.3%	1	15.8%	2
0.7	2.8%	1	12.3%	1
0.8	0.3%	1	9.8%	1
0.9	-2.3%	0	7.3%	1

This table shows how the greater savings in the drone stop cost when $s_d = -0.2$ leads to larger overall savings with a maximum of 56.7% (versus 39.8% when $s_d = 0$) and greater use of multiple drones per truck even with relatively large drone operating costs. Similar results to Figures 5 and 6 and Table 1 are produced using other delivery densities, because higher drone operating costs per mile lead to fewer drones per truck and lower savings per stop.

5.2 Impact of Density of Deliveries

This section explores the impact of the density of deliveries in a region. As a baseline, we consider $s_d = -0.1$ (the drone stop cost is 10 cents *less* than the truck stop cost). Figure 8 shows how truck-drone delivery savings per delivery ($PSAV$) decrease at a decreasing rate as customer density increases for selected values of the drone operating cost.
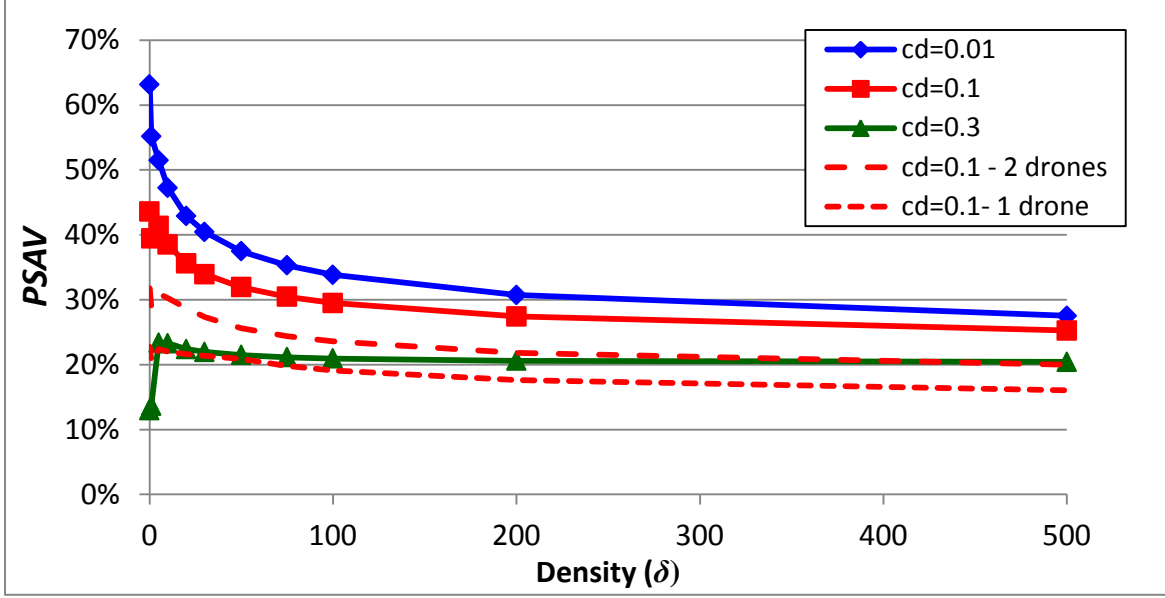
Figure 8. Percentage savings with one to eight drones per truck with increasing delivery density for $s_d = -0.1$

Each solid curve for the different drone operating costs allows the use of up to eight drones per truck whichever number provides the lowest cost. Also, the two dashed curves for one and two drones per truck when $c_d = 0.1$ highlight the benefits of allowing multiple drones per truck. In this figure, the leftmost density values can be viewed as sparsely populated rural areas ($\delta = 0.01$ is one delivery per 100 mi^2), with the central and right-most values more typical of suburban regions in the US. (The average housing unit density in the US is 32.8 per mi^2 overall and 128.7 per mi^2 in metropolitan areas [Census-Charts.com 2017].) With very low drone operating costs ($c_d = 0.01$; the upper curve in Figure 7), truck-drone delivery provides savings of up to 63.2% per delivery for very low densities of deliveries. Savings decrease with the drone operating costs, but even for densities of 500 per mi^2, the savings range from 20% to 27.5% for $0.01 \le c_d \le 0.3$. For the solid lines in this figure, the number of drones per truck is the value that provides the lowest cost (and thus the greatest $PSAV$ values). This is always eight drones per truck for $c_d = 0.01$ and $c_d = 0.1$, but it varies from two to seven drones per truck for $c_d = 0.3$ depending on the delivery density. Limiting the number of drones per truck reduces $PSAV$. When $c_d = 0.1$, allowing only one drone per truck produces savings per delivery of only 22.8% for $\delta = 0.01$ and 16.1% for $\delta = 500$ versus 43.6% and 25.3%, respectively, when allowing up to eight drones per truck.

The large percentage savings per delivery ($PSAV$) with low delivery densities suggest a high value from using drones for delivery in rural areas; however, the low density of deliveries in rural areas means that these large savings apply to relatively few deliveries. The savings per square mile ($SPSM$) combines $PSAV$ with the density of deliveries and the expected cost per delivery (which varies with delivery density):

$$SPSM = PSAV \times \delta \times E_{to} = [E_{to} - E_{td}(n)] \times \delta. \tag{29}$$

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$SPSM$ is the spatial density of total delivery savings, and it provides a very different perspective than the savings per delivery ($PSAV$). Figure 9 provides $SPSM$ for the same situations depicted in Figure 8 and shows how the intensity of savings increases with the delivery density.
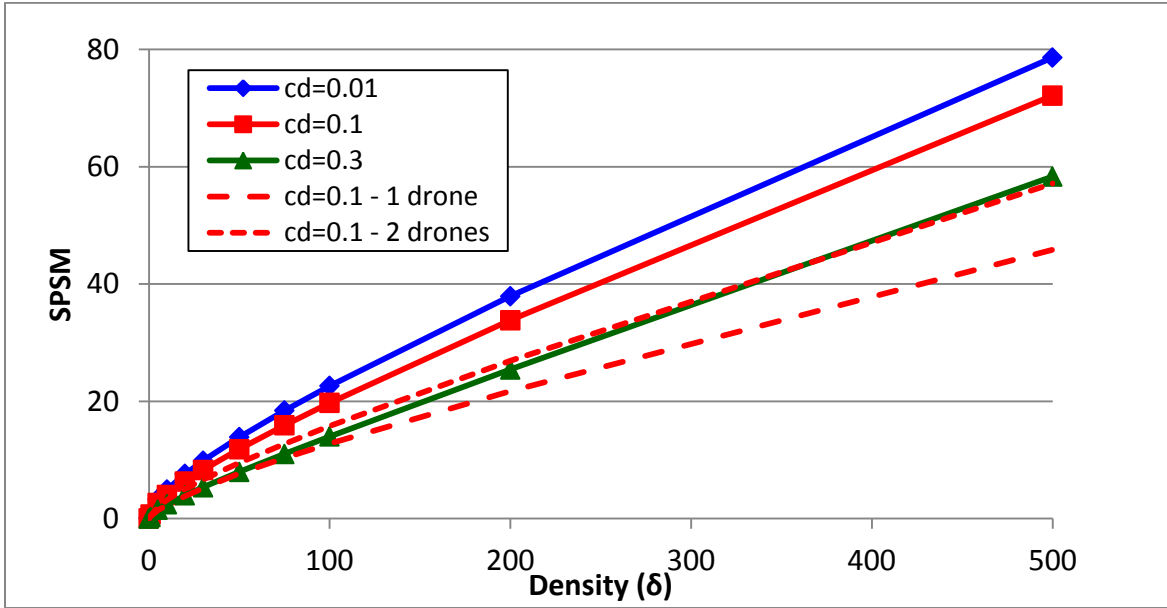

Figure 9. Savings per square mile with one to eight drones per truck with increasing delivery density for $s_d = -0.1$

This figure better shows the important and growing benefits from using multiple drones per truck with large delivery densities. For example, when $\delta = 500$ and $c_d = 0.1$, $SPSM$ is \$45.9/mi^2 with one drone per truck, \$57.1/mi^2 with two drones per truck and \$72.1/mi^2 with eight drones per truck.

Even with expensive drones where $c_d = 0.625$ (half the cost per mile for trucks), when $s_d = -0.1$, $SPSM$ is \$35.8/mi^2 for $\delta = 500$ and \$15.4/mi^2 for $\delta = 200$, though it is only \$0.42/mi^2 for $\delta = 5$. With this large drone operating cost and even lower delivery densities, $SPSM$ becomes negative because truck-only delivery provides lower costs due to the differences in total distance traveled for truck-drone and truck-only delivery and the interplay between linehaul costs and the number of deliveries per route. Because hybrid truck-drone routes require longer total travel distance for the truck and drone together compared to truck-only routes, though less distance for the truck travel, as shown in Figure 5, a high drone operating cost per mile (relative to the truck cost per mile) causes truck-drone delivery to have higher total costs than truck-only delivery. However, with higher delivery densities, there are more deliveries per route and linehaul travel becomes an important component of the total cost per delivery. This has a greater impact for truck-only travel because those routes have fewer deliveries than truck-drone routes.

We limit our presentation of results to a maximum of $\delta = 500$, though the models apply for even greater densities. However, a very high delivery density is likely to occur in a dense urban

region where practical considerations reduce the appropriateness of the model for travel and for delivery.

While Figures 8 and 9 are constructed for a marginal drone stop cost of $s_d = -0.1$, similar results for $PSAV$ and $SPSM$ with $s_d = 0$ are shown in Figures 10 and 11.
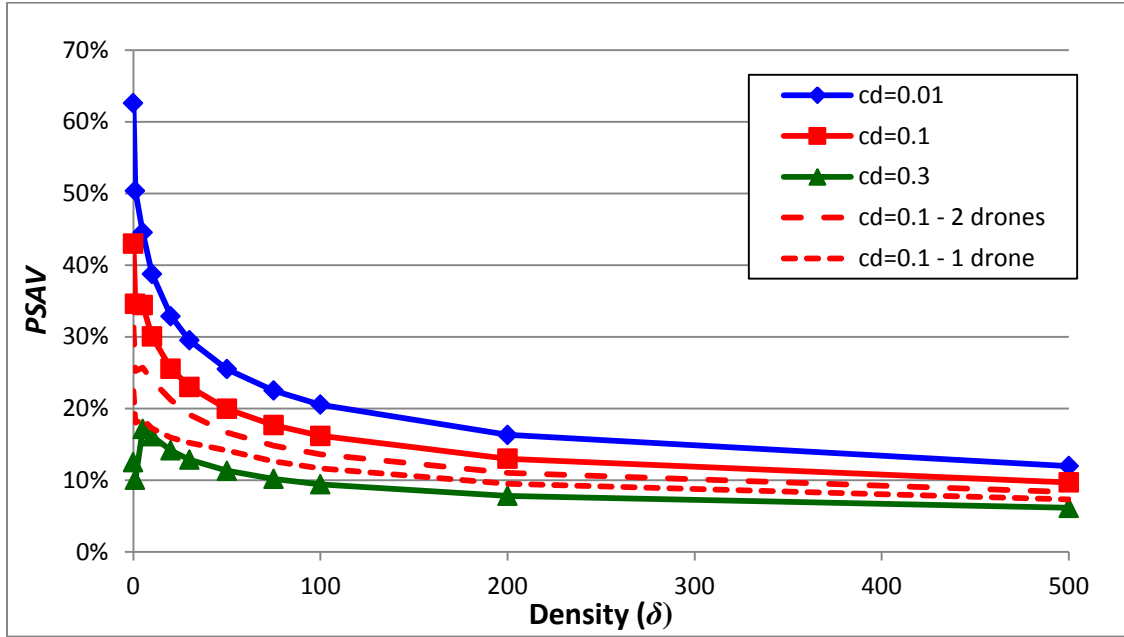

Figure 10. Percentage savings per delivery for one to eight drones per truck with increasing delivery density for $s_d = 0$
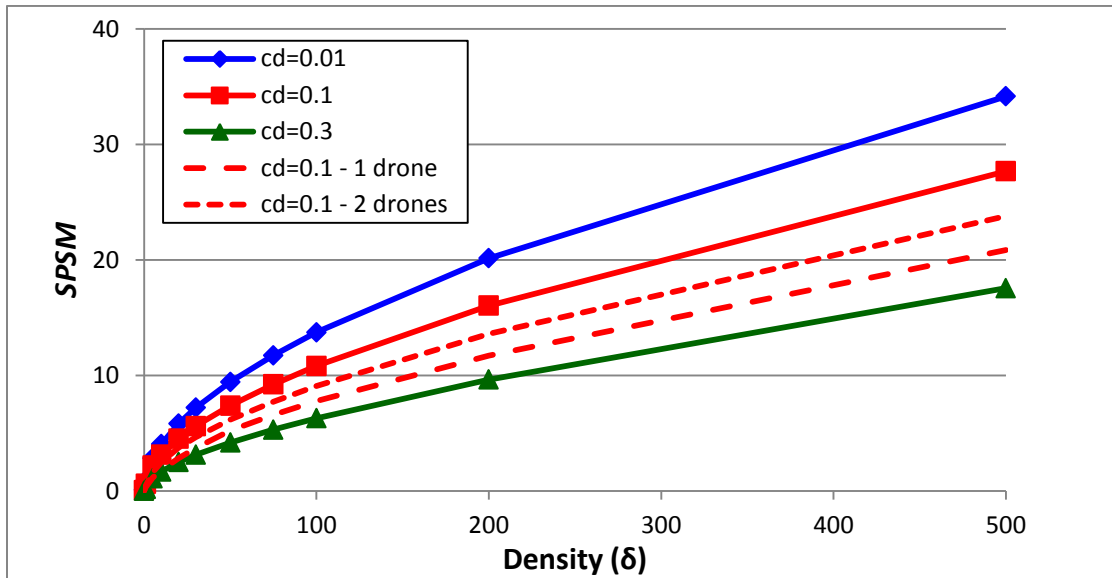

Figure 11. Savings per square mile with one to eight drones per truck with increasing delivery density for $s_d = 0$

With very low delivery densities, the results for $s_d = 0$ are very similar to those for $s_d = -0.1$ because there are so few deliveries that the stop cost s_d is a small component of total costs. However, with increasing delivery density, the stop cost becomes important and the stop cost savings with $s_d = -0.1$ makes drone use more attractive. For example, in Figure 8 with $s_d = -0.1$ and $\delta = 500$, $PSAV$ ranges from 20.0% to 27.5% for $0.01 \leq c_d \leq 0.3$, while in Figure 10 with $s_d = 0$, $PSAV$ ranges from 6.1% to 12.0%. Even one drone per truck provides much greater savings when $s_d = -0.1$, because the savings with $\delta = 500$ and $c_d = 0.1$ are 16.1% versus 7.3% for $s_d = 0$. The benefits from the reduction in marginal drone stop cost are also clear from comparing Figures 9 and 11, where the savings intensity is much larger when $s_d = -0.1$. Further, the relative benefits of $s_d = -0.1$ (Figure 9) versus $s_d = 0$ (Figure 11) increase with the delivery density. The results when $s_d = 0$ reflect the use of eight drones per truck for low values of drone operating cost ($c_d \leq 0.1$) but only two drones per truck for $c_d = 0.3$, unlike the results when $s_d = -0.1$, where the number of drones per truck increases with the delivery density. Such an increase does not happen with $s_d = 0$ because drone use is not as attractive as when there are drone stop cost savings.

5.3 Impact of Marginal Drone Delivery Cost

To illustrate the impact of the marginal drone delivery cost on the savings from truck-drone delivery, Figure 12 shows $PSAV$ with up to eight drones per truck for $c_d = 0.1$ and four selected values of the drone marginal delivery cost: $s_d = $ -0.2, -0.1, 0, and 0.1.
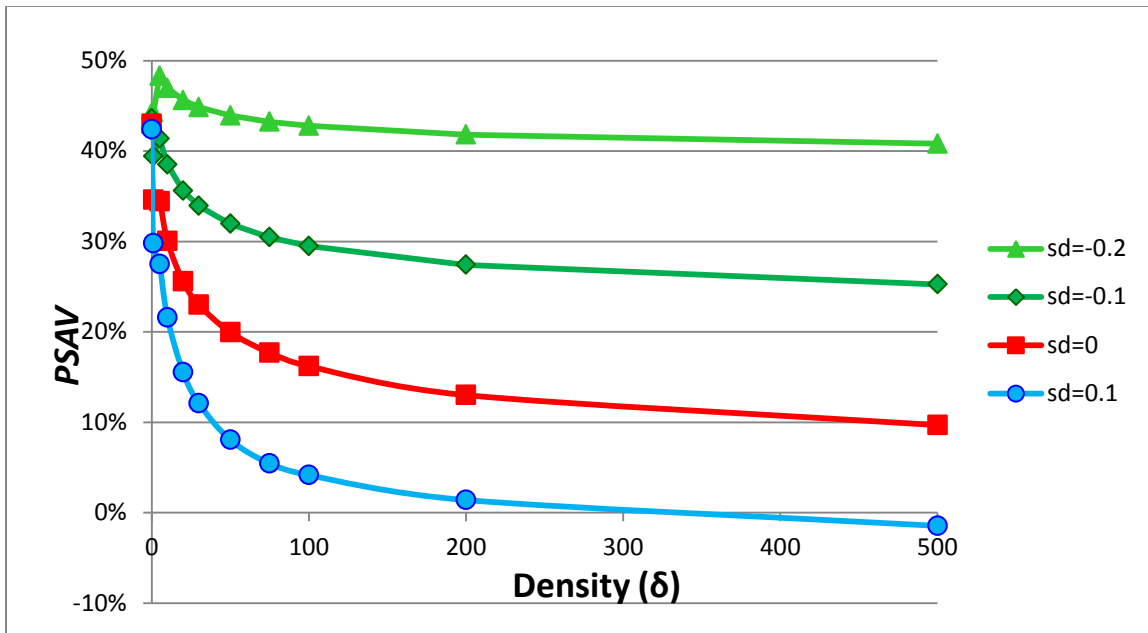

Figure 12. Percentage savings per delivery with up to eight drones per truck with increasing delivery density for $c_d = 0.1$

With a large drone marginal stop cost savings of $s_d = -0.2$, truck-drone delivery provides large savings per delivery (about 40% to 50%) that are relatively insensitive to the delivery density

beyond the smallest densities. Conversely, when the drone stops are more expensive than truck stops ($s_d = 0.1$), the benefits per delivery of truck-drone delivery are very sensitive to the delivery density and decrease dramatically, even becoming negative (i.e., truck-only delivery is better) for densities larger than about 300 per square mile. Table 2 summarizes $PSAV$ results with one, two, and up to eight drones per truck for $c_d = 0.1$ and $c_d = 0.01$ with four levels of marginal drone stop cost in both "rural" regions (i.e., $\delta \leq 10$ /mi^2) and "suburban" regions (i.e., $30 \leq \delta \leq 500$ /mi^2).

Table 2. Percentage savings ($PSAV$) and number of drones per truck (n) with different drone marginal stop costs for $c_d = 0.1$

n	s_d	$c_d = 0.1$		$c_d = 0.01$	
			Suburban		Suburban
		Rural $\delta \leq 10$	$\delta = 30 - 500$	Rural $\delta \leq 10$	$\delta = 30 - 500$
1	-0.2	$23.1 - 26.8\%$	$24.8 - 27.5\%$	$28.6 - 29.2\%$	$25.4 - 29.3\%$
	-0.1	$22.0 - 22.8\%$	$16.1 - 21.4\%$	$24.4 - 28.2\%$	$16.7 - 23.2\%$
	0	$17.2 - 22.4\%$	$7.3 - 15.2\%$	$19.7 - 27.9\%$	$7.9 - 17.0\%$
	0.1	$12.5 - 22.1\%$	$-1.5 - 9.1\%$	$14.9 - 27.5\%$	$-0.8 - 10.9\%$
2	-0.2	$32.2 - 36.6\%$	$31.7 - 35.5\%$	$40.6 - 41.0\%$	$32.7 - 38.4\%$
	-0.1	$30.3 - 31.8\%$	$20.0 - 27.3\%$	$34.2 - 40.6\%$	$21.0 - 30.2\%$
	0	$23.9 - 31.3\%$	$8.3 - 19.1\%$	$27.9 - 40.1\%$	$9.4 - 22.1\%$
	0.1	$17.6 - 30.9\%$	$-3.3 - 10.9\%$	$21.5 - 39.7\%$	$-2.3 - 13.9\%$
1-8	-0.2	$44.2 - 47.0\%$	$40.8 - 44.8\%$	$55.7 - 63.8\%$	$43.1 - 53.4\%$
	-0.1	$38.5 - 43.6\%$	$25.3 - 33.9\%$	$47.3 - 63.2\%$	$27.5 - 40.4\%$
	0	$30.0 - 43.0\%$	$9.7 - 23.0\%$	$38.8 - 62.6\%$	$12.0 - 29.5\%$
	0.1	$21.6 - 42.4\%$	$-1.5 - 12.1\%$	$30.0 - 62.0\%$	$-0.8 - 18.6\%$

The first two columns provide the number of drones per truck and the marginal drone stop cost. The next four columns provide the range of savings ($PSAV$) in the rural and suburban regions for the two values of the drone operating cost per mile. This table shows greater savings as the drone marginal stop cost decreases (as expected) and the benefits of using multiple drones per truck. With a positive marginal drone stop cost of $s_d = 0.1$ (drone stop cost is 10 cents *more* than the truck stop cost), drones are naturally less attractive and $PSAV$ may become negative (i.e., truck-only delivery is preferred) with high delivery densities. However, Table 2 shows that in rural areas the savings from hybrid truck-drone delivery are positive even with a positive marginal drone stop cost of $s_d = 0.1$, showing that truck-drone delivery is preferred over truck-only delivery at these low densities.

Figure 13 is a companion to Figure 12 that presents the $SPSM$ for the situations depicted in Figure 12.
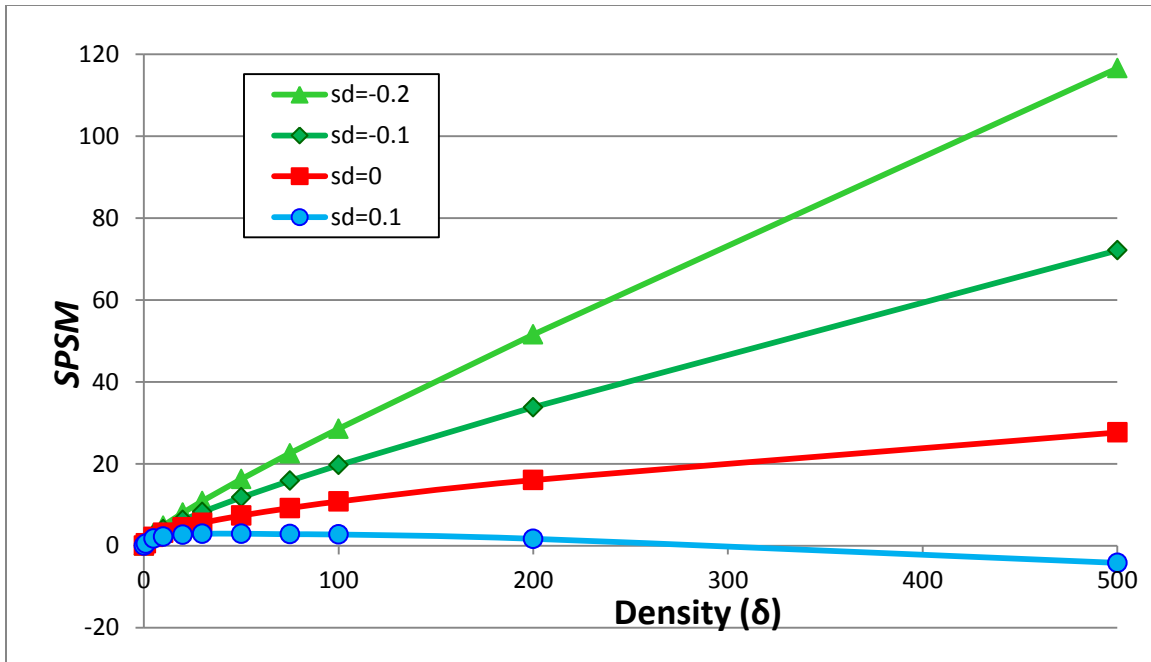
Figure 13. Savings per square mile with one to eight drones per truck with increasing delivery density for $c_d = 0.1$

Given the relatively flat curves in Figure 12 for moderately large delivery densities, Figure 13 shows nearly linear curves reflecting the greater benefits in regions of higher density. The strong advantages of reduced marginal stop costs are more evident in the growing separation of the curves in Figure 13 versus the nearly parallel lines in Figure 12. Figure 13 also shows the low savings (or small losses) from truck-drone delivery when drone deliveries are more expensive than truck deliveries.

Collectively, the results highlight that truck-drone delivery appears to have strong potential to reduce delivery costs. Regions of low delivery density often provide the greatest percentage savings per stop, but these savings apply to relatively few deliveries. The net savings per square mile generally increase with the delivery density, so regions of greater delivery density, which may have a lower savings per delivery, actually often generate much greater total savings.

6. CONCLUSIONS

This research derived continuous approximation models for travel cost and travel time of hybrid truck-drone delivery and truck-only delivery to facilitate a strategic analysis of the design of drone delivery systems. The mathematical expressions in the models are optimized to determine important drone delivery system design parameters, including the optimal number of truck and drone deliveries per route, the optimal number of drones per truck, and whether truck-only or hybrid truck-drone travel provides a lower total cost. Travel time constraints and vehicle capacity limits are incorporated in the models to increase their realism. The CA models allow us to assess the best use of truck-drone delivery, the appropriate mix of drones and trucks, and the expected economic performance of different delivery systems. Comparison of the expected delivery costs from the CA models for different delivery systems helps identify when and where implementing truck-drone delivery is likely to be beneficial.

The results highlight some key tradeoffs in the use of the trucks and drones and their dependence on relative operating characteristics, costs, and delivery densities. Key findings are as follows:

- Truck-drone hybrid delivery has the potential to provide substantial cost savings, especially in suburban areas.

- Incorporating multiple drones per truck offers important but marginally decreasing savings that can be large.

- The benefits from truck-drone delivery depend strongly on the relative operating costs per mile for trucks and drones, the relative stop costs for trucks and drones, and the spatial density of customers.

- Measures of savings per delivery and savings intensity per square mile provide complementary perspectives that highlight the conditions and regions likely to generate the greatest savings.

- The importance of drone operating costs is highlighted in the results, and it is likely these costs will continue to fall as drone technology advances. However, more detailed analysis to identify real values of cost per mile and per stop would be very useful. The continuous approximation models provide valuable managerial insights by analyzing a general version of the delivery problem, and a wide range of different settings can be modeled and analyzed by varying the delivery (demand) density and the operating characteristics for drones and trucks.

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