A modified traffic fundamental diagram for the mixed autonomy highway traffic flow

Mingfeng Shang, Raphael Stern
shang140@umn.edu

Presentation at the 2022 Mid-Continent Transportation Research Symposium

09/14/2022
Phantom traffic jams: real jams that happen for no apparent reason – observed in the field, recreated in the lab.
Phantom traffic jams: result of unstable traffic

String unstable traffic

Small perturbations from the equilibrium spacing will amplify as the propagate along the platoon of vehicles

String stable traffic

Small perturbations from the equilibrium spacing will dissipate as the propagate along the platoon of vehicles

[Wilson and Ward, 2010]
A single, individual AV may dampen traffic waves

• When traffic is fully human-driven:
  • Traffic waves emerge

• When there are some AVs in the traffic
  • Traffic waves can be dampened with even a single AV

• However, not all AVs are the same…

[Stern, 2017; Sugiyama, et al. 2008]
## Level of automation (Level 0 – Level 5)

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Intervention when needed</th>
<th>Robot in control</th>
<th>Impacts on Traffic flow</th>
<th>Motivation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>No automation</td>
<td>✓</td>
<td>×</td>
<td>Baseline</td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td>Driver assistance</td>
<td>✓</td>
<td>×</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Level 2</td>
<td>Partial automation</td>
<td>✓</td>
<td>×</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>Conditional automation</td>
<td>✓</td>
<td>×</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Level 4</td>
<td>High automation</td>
<td>×</td>
<td>×</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Level 5</td>
<td>Full automation</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

**Motivation:** How will partially automated vehicles influence traffic flow?

- Increase string stability
- Increase throughput
- Reduce fuel consumption

[Source: Society of Automotive Engineers, 2018; Talebpour et al., 2016; Qin et al., 2018]
Microscopic car following (CF) model

• To study ACC driving behavior, first need framework to model traffic flow

• Model car following dynamics (Physics-based CF model):

\[
\ddot{x}_j(t) = f(s_j(t), \dot{s}_j(t), \dot{x}_j(t))
\]

  Acceleration of vehicle \( j \)
  Space in front of vehicle \( j \)
  Speed of vehicle \( j \)

  Relative speed in front of vehicle \( j \)

• Can be used for traffic simulation and analysis

[Gipps, 1956; Treiber, Hennecke, Belbing, 2000; Bando 1996; Mo, et al. 2021; ]
Commonly-used CF model:

Optimal velocity relative velocity model (OVRV)

- Common assumption: constant time gap control law
- Used to capture ACC driving behavior in [Gunter et al. 2020] and [de Souza and Stern 2020] with low simulation errors

\[
\ddot{x}(t) = k_1 \left[ s - (\eta + \tau \dot{x}) \right] + k_2 \dot{s}
\]

Intelligent driver model (IDM)

\[
\ddot{x} = a \left( 1 - \left( \frac{\dot{x}}{v_0} \right)^\delta - \left( \frac{\dot{s}(\dot{x}, \ddot{s})}{s} \right)^2 \right)
\]

\[
\dot{s}(\dot{x}, \ddot{s}) = \eta + \tau \dot{x} - \frac{\dot{x} \ddot{s}}{2\sqrt{ab}}
\]

\[a, b: \text{maximum acceleration and comfortable braking rate}
\]

\[v_0: \text{desired limit speed}
\]

\[s_0: \text{jam distance (minimum distance)}
\]

\[\tau: \text{time-gap (headway)}
\]

\[\delta: \text{acceleration exponent}
\]

What does this mean for traffic flow?

- Simulate traffic flow using calibrated models on a sample roadway
- Compare different vehicles using calibrated microscopic vehicle models:
  - Commercial ACC Vehicle
  - Fully automated AV
- Consider bottleneck capacity at different automated vehicle market penetration rates
"The speeds show an unusual traffic condition to have existed. Apparently most of the drivers were more interested in looking at the cattle than they were in reaching their destinations"
Traffic fundamental diagram: a flow-density relationship

- A flow-density diagram (1930s)

- Maximum flow obtained at the critical density
Fully automated vehicles may increase capacity (Simulation)

- Simulate traffic flow using calibrated models on a sample roadway

- Fully automated vehicles may increase capacity

---

Increasing autonomy

### 0% AV

- Flow (veh/hr)
- Density (veh/km)

### 100% AV

- Flow (veh/hr)
- Density (veh/km)
…but ACC vehicles means lower capacity

0% ACC (all human)  
20% ACC  
40% ACC  
60% ACC  
80% ACC  
100% ACC
Commercially-available ACC vehicles may reduce bottleneck capacity

• Bottleneck capacity at Detector 1 increases at higher fully AV market penetration rates

• Commercially available ACC vehicles generally decrease capacity

• Up to a 35% reduction in capacity as a result of commercially available ACC vehicles
Assume human drivers drive according to the IDM

\[ \ddot{x} = a_h \left( 1 - \left( \frac{v}{v_{0h}} \right)^{\delta_h} - \left( \frac{\dot{s}(v, \dot{s})}{s} \right)^2 \right) \]

\[ \dot{s}(v, \dot{s}) = s_{0h} + \tau_h v - \frac{v \dot{s}}{2 \sqrt{a_h b_h}} \]

At equilibrium,

\[ 0 = a_h \left( 1 - \left( \frac{v}{v_{0h}} \right)^{\delta_h} - \left( \frac{s_{0h} + \tau_h v}{s} \right)^2 \right) \]

In congestion,

\[ \rho = \frac{1}{s + l_c} = \frac{1}{\frac{s_{0h} + \tau_h v}{\sqrt{1 - \left( \frac{v}{v_{0h}} \right)^{\delta_h}}} + l_c} \]

\[ q = \rho v = \frac{v}{\frac{s_{0h} + \tau_h v}{\sqrt{1 - \left( \frac{v}{v_{0h}} \right)^{\delta_h}}} + l_c} \]

[Treiber, 2013]
Deriving a mixed autonomy fundamental diagram

- Resulting IDM fundamental diagram (for human drivers, IDM+):

\[
q(\rho) = \begin{cases} 
  u_f \rho, & \rho \leq \rho_{cr}, \\
  \frac{1 - \rho(l_c + s_{0h})}{\tau_h}, & \rho > \rho_{cr}.
\end{cases}
\]

- For automated vehicles (e.g., ACC), use OVM:

\[
\ddot{x}(t) = k_{1a} (s - \eta_a - \tau_a v) + k_{2a} \dot{s}
\]

- At equilibrium:

\[
0 = k_{1a} (s - \eta_a - \tau_a v) \quad \rightarrow \quad s = \eta_a + \tau_a v \quad \rightarrow \quad q(\rho) = \frac{1 - \rho (l_c + \eta_a)}{\tau_a}
\]

- Resulting fundamental diagram:

\[
q(\rho) = \begin{cases} 
  u_f \rho, & \rho \leq \rho_{cr}, \\
  \frac{1 - \rho (l_c + \eta_a)}{\tau_a}, & \rho > \rho_{cr}.
\end{cases}
\]
Combining individual species fundamental diagrams

• Uncongested flow: Vehicles in free flow so flow depends on density since AVs and human-driven vehicles are at the same speed

• Congested flow – compute equivalent spacing:

\[
\bar{s} = \gamma \bar{s}_a + (1 - \gamma) \bar{s}_h = \gamma \left( \eta_a + \tau_a v \right) + (1 - \gamma) \frac{s_{0h} + \tau_h v}{\sqrt{1 - \left( \frac{v}{v_{0h}} \right) \frac{\delta_h}{\delta}}}
\]

• When human-driven vehicle speed less than desired, simplification:

\[
\bar{s} = \gamma \left( \eta_a + \tau_a v \right) + (1 - \gamma) \left( s_{0h} + \tau_h v \right)
\]

• Corresponding mixed-flow density:

\[
\bar{\rho} = \frac{1}{\bar{s} + l_c} = \frac{1}{\gamma \left( \eta_a + \tau_a v \right) + (1 - \gamma) \left( s_{0h} + \tau_h v \right) + l_c}.
\]
Combined two-species fundamental diagram

- Combined fundamental diagram:

\[ q(\bar{\rho}, \gamma) = \begin{cases} u_f \bar{\rho}, & \bar{\rho} \leq \bar{\rho}_{cr} \\ \frac{1-\bar{\rho}(l_c+\gamma \eta_a+(1-\gamma) s_0 h)}{\gamma \tau_a+(1-\gamma) \tau_h}, & \bar{\rho} > \bar{\rho}_{cr} \end{cases} \]

- Plotting with calibrated values for automated vehicles and human drivers
Future work: Actuate control infrastructure based on vehicles in the flow

Identified market penetration rates → Vehicle dynamics → Model aggregate flow → Aggregate flow dynamics → Design extending ramp metering controller

- Identified market penetration rates
- Vehicle dynamics
- Model aggregate flow
- Aggregate flow dynamics
- Design extending ramp metering controller

Human-driven vehicle
Automated vehicle
Control signal
Controlling mainline highway flow via ramp metering

- Ramp metering used to regulate flow of traffic into merge bottleneck

- General goal: maintain at critical density to maximize throughput and minimize waiting time

- Twin Cities ramp metering experiment: up to 55% reduction in fuel consumption

- How will ramp metering work with mixed autonomy traffic flow?
A modified traffic fundamental diagram for the mixed autonomy highway traffic flow

Mingfeng Shang, Raphael Stern
shang140@umn.edu

Presentation at the 2022 Mid-Continent Transportation Research Symposium

09/14/2022
Intelligent driver model (IDM)

\[ \ddot{x} = a \left( 1 - \left( \frac{\dot{x}}{v_0} \right)^\delta - \left( \frac{\dot{s}(\dot{x}, \dot{s})}{s} \right)^2 \right) \]

\[ \dot{s}(\dot{x}, \dot{s}) = \eta + \tau \dot{x} - \frac{\dot{x} \dot{s}}{2\sqrt{ab}} \]

- \( a, b \): maximum acceleration and comfortable braking rate
- \( v_0 \): desired limit speed
- \( s_0 \): jam distance (minimum distance)
- \( \tau \): time-gap (headway)
- \( \delta \): acceleration exponent

Parameter values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Human driver</th>
<th>ACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_h )</td>
<td>maximum acceleration (m/s²)</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>( b_h )</td>
<td>comfortable deceleration (m/s²)</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>( v_{0,h} )</td>
<td>desired speed (m/s)</td>
<td>33.3</td>
<td>-</td>
</tr>
<tr>
<td>( s_{0,h} )</td>
<td>jam spacing (m)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>( \delta_h )</td>
<td>exponential component</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>( \tau_h )</td>
<td>time gap (s)</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>( k_{1e} )</td>
<td>velocity gain</td>
<td>-</td>
<td>0.02</td>
</tr>
<tr>
<td>( k_{2e} )</td>
<td>relative velocity gain</td>
<td>-</td>
<td>0.13</td>
</tr>
<tr>
<td>( \eta_h )</td>
<td>jam distance (m)</td>
<td>-</td>
<td>21.51</td>
</tr>
<tr>
<td>( \tau_a )</td>
<td>time gap (s)</td>
<td>-</td>
<td>1.71</td>
</tr>
<tr>
<td>( l_e )</td>
<td>car length (m)</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

[Treiber, et al, 2000]
Task 3.1: Model aggregate flow for mixed autonomy traffic

- Goal: model density and market penetration rate in merge section

- Two-species cell transmission model:

\[ n^k_i = n_{i-1}^k + (y^k_{i-1} + y^k_r - y^k_i) \]

Evolution of all vehicles

\[ n^k_{AV,i} = n_{AV,i-1}^k + (y^k_{AV,i-1} + y^k_{AV,r} - y^k_{AV,i}) \]

Evolution of AVs

- Challenge: compute mixed autonomy boundary flows

- Update market penetration rate with evolution of all vehicles and AVs

[Daganzo, 1994; Levin, 2016]
Task 3.2: Extending ramp metering control to mixed autonomy traffic flow with varying degrees of automation

- ALINEA ramp metering control

\[ r(k) = r(k - 1) + K_R \left[ \rho_{cr} - \rho(k) \right] \]
Education

• **University of Minnesota, Twin cities (2019 – Present)**
  • Ph.D. student in Civil Engineering

• **University of Illinois, Urbana-Champaign (2016 – 2018)**
  • M.S. in Civil Engineering

• **Southwest Jiaotong University, Chengdu (2013 – 2016)**
  • B.E. in Civil Engineering
Outline: modeling and control mixed autonomy traffic

• Task 1: Accurately modeling vehicle dynamics
  – Modeling asymmetric ACC driving behavior ✓
  – Modeling human driver stochastic driving behavior ✓
  – Improvement of modeling ACC vehicles and planning on higher-level automated vehicles (Summer 22)

• Task 2: Assess the impacts of mixed autonomy traffic flow
  – String stability ✓
  – Highway throughput and fundamental diagram ✓
  – Fuel consumption and emissions ✓

• Task 3: Traffic control in the presence of mixed autonomy traffic
  – Macroscopic on-ramp traffic flow dynamics for mixed autonomy traffic (Summer 22)
  – Extending ramp metering control to mixed autonomy traffic flow (Fall 22)
  – Development of advanced ramp metering strategies (Spring 23)
Theme: modeling and control of mixed autonomy traffic

1) Accurately modeling vehicle dynamics

2) Assess the impacts of mixed autonomy traffic flow

3) Control the results for better performance

Human-driven vehicle

Commercially available ACC vehicle

Infrastructure control

String stability

Throughput

Fuel consumption and emissions

Mixed autonomy flow
Task 1: Accurately modeling vehicle dynamics *(In progress)*

- Task 1.1: Modeling asymmetric ACC driving behavior *(Completed)*
- Task 1.2: A continuous asymmetric car following model considering vehicle actuation delay *(In progress, Summer 22)*
- Task 1.3*: Modeling human driver stochastic driving behavior *(Completed)*

[Shang et al., 2022, Shang and Stern, 2020]
Challenge: symmetric behavior unrealistic

- Standard models assume similar braking and acceleration behavior

- Braking and acceleration are two fundamentally different mechanisms on vehicles
  - Cars can typically brake much faster than they can accelerate

- Results in under predicting braking and over predicting acceleration
Proposed model: the Asymmetric OVRV (AOVRV) model

The AOVRV model:

\[
\ddot{x}(t) = \begin{cases} 
    f_{\text{Decel}}(\theta_D, s(t), \dot{x}(t), \dot{s}(t)) & \dot{s}(t) \leq 0 \\
    f_{\text{Accel}}(\theta_A, s(t), \dot{x}(t), \dot{s}(t)) & \dot{s}(t) > 0.
\end{cases}
\]

or written as:

\[
\ddot{x}(t) = k_{1D}\tau (V_D(s) - \dot{x}) \Theta(-\dot{s}) + k_{2D}\Theta(-\dot{s})\dot{s} + k_{1A}\tau (V_A(s) - \dot{x}) \Theta(\dot{s}) + k_{2A}\Theta(\dot{s})\dot{s}
\]

\[
V_A(s) = V_D(s) = \frac{s - \eta}{\tau} \\
\Theta(\dot{s}) = \begin{cases} 
    0 & \dot{s} \leq 0 \\
    1 & \dot{s} > 0
\end{cases}
\]

Recall the OVRV model:

\[
\ddot{x} = f(s, \dot{s}, \dot{x}) = k_1 (s - \eta - \tau \dot{x}) + k_2 \dot{s}
\]

AOVRV and OVRV: \(\tau\) and \(\eta\) are the same
Test broad range of vehicles with oscillatory test

- Seven vehicles are tested
- Testing at the minimum following setting and the maximum following setting

---

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Make</th>
<th>Style</th>
<th>Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>Full-size sedan</td>
<td>Combustion</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>Compact sedan</td>
<td>Combustion</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>Compact hatchback</td>
<td>Hybrid</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>Compact SUV</td>
<td>Combustion</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>Compact SUV</td>
<td>Combustion</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>Mid-size SUV</td>
<td>Combustion</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
<td>Full-size SUV</td>
<td>Combustion</td>
</tr>
</tbody>
</table>

[Graph showing oscillatory test results]

[Gunter et al. 2020]
Microscopic model calibration: minimize spacing RMSE

**RMSE:**

\[
\text{minimize} : \quad \sqrt{\frac{1}{T} \sum_{t=0}^{T} (s_m(t) - s(t))^2}
\]

subject to:

\[
\begin{align*}
\dot{v}(t) &= f(\theta, s, \dot{s}, \dot{x}) \\
\dot{s}(t) &= \dot{x}_{\ell,m}(t) - \dot{x}(t) \\
s(0) &= s_m(0) \\
\dot{x}(0) &= \dot{x}_m(0) \\
\theta &\in \theta_f
\end{align*}
\]

\( m \) represents measured quantities,
\( \theta_f \) represents a boundary constraint of parameter values,
\( T \) is the duration time in the dataset being used for calibration.

**Simulation process:**

\[
\begin{bmatrix}
\dot{x} \\
\dot{x}
\end{bmatrix}_{t+\Delta t} =
\begin{bmatrix}
\dot{x} \\
\dot{x}
\end{bmatrix}_t +
\begin{bmatrix}
\dot{x} \\
f(\theta, s, \dot{x}, \dot{s})
\end{bmatrix}_t \Delta t
\]
Microscopic model calibration

Algorithm: Training model parameters to obtain the best-fit model parameter values
1. set input data, initial model parameter, number of iterations
2. for iteration = 1: number of iterations
3. obtain the simulated vehicle trajectory \( M(\theta) \) with the current parameter \( \theta \)
4. update \( \theta \) by minimizing \( \epsilon(\theta) \) with the interior-point nonlinear optimization algorithm
5. end

Simulated velocity with the AOVRV is closer to the field data

Two-vehicle experiment

Simulated velocity:

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>( k_{1A} )</th>
<th>( k_{1D} )</th>
<th>( k_{2A} )</th>
<th>( k_{2D} )</th>
<th>( \tau )</th>
<th>RMSE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-min</td>
<td>0.017</td>
<td>0.039</td>
<td>0.158</td>
<td>0.393</td>
<td>0.131</td>
<td>1.917</td>
</tr>
<tr>
<td>A</td>
<td>0.0353</td>
<td>0.0645</td>
<td>2.78</td>
<td>0.113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.0704</td>
<td>0.157</td>
<td>1.41</td>
<td>0.0489</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.0169</td>
<td>0.123</td>
<td>2.50</td>
<td>0.0600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.0379</td>
<td>0.140</td>
<td>1.57</td>
<td>0.0751</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.0225</td>
<td>0.107</td>
<td>2.84</td>
<td>0.0655</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.0512</td>
<td>0.0945</td>
<td>1.49</td>
<td>0.0810</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.0281</td>
<td>0.116</td>
<td>2.71</td>
<td>0.0679</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.0583</td>
<td>0.0958</td>
<td>1.54</td>
<td>0.0539</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.0666</td>
<td>0.0261</td>
<td>2.36</td>
<td>0.0365</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.0848</td>
<td>0.0652</td>
<td>1.42</td>
<td>0.0686</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.0447</td>
<td>0.0615</td>
<td>2.25</td>
<td>0.0578</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.0803</td>
<td>0.0657</td>
<td>1.46</td>
<td>0.0647</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.0472</td>
<td>0.0584</td>
<td>2.24</td>
<td>0.0482</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[Gunter et al. 2019]
Initial guess $\theta$

Data

Simulation with (13)

RMSE < threshold?

Update $\theta$

No

Yes

The best-fit $\theta$
Significance test to compare the AOVRV model to other microscopic car following models

The spacing RMSE percentage change:

$$\psi_k = \frac{\text{RMSE}_{\text{AOVRV}} - \text{RMSE}_{k}}{\text{RMSE}_{k}} \times 100\%$$

The sum of squared spacing error (SSE):

$$\text{SSE} = \sum_{t=0}^{T} (s_m(t) - s(t))^2$$

F-statistics to estimate p-value:

$$F_k = \frac{\text{SSE}_{\text{AOVRV}}}{\text{SSE}_{k}}$$

<table>
<thead>
<tr>
<th>Vehicle-setting</th>
<th>Significance test for the OVRV</th>
<th>Significance test for the IDM</th>
<th>Significance test for the AFVD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_{\text{OVRV}}$ [%]</td>
<td>$P_{\text{OVRV}}$</td>
<td>$\psi_{\text{IDM}}$ [%]</td>
</tr>
<tr>
<td>A-min</td>
<td>-3.22</td>
<td>0.956</td>
<td>AOVRV</td>
</tr>
<tr>
<td>A-max</td>
<td>-0.34</td>
<td>0.570</td>
<td>Same</td>
</tr>
<tr>
<td>B-min</td>
<td>-38.03</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>B-max</td>
<td>-10.38</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>C-min</td>
<td>-0.40</td>
<td>0.582</td>
<td>Same</td>
</tr>
<tr>
<td>C-max</td>
<td>-30.26</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>D-min</td>
<td>-34.91</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>D-max</td>
<td>-31.96</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>E-min</td>
<td>-2.37</td>
<td>0.895</td>
<td>Same</td>
</tr>
<tr>
<td>E-max</td>
<td>-7.66</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>F-min</td>
<td>-7.94</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>F-max</td>
<td>-6.91</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>G-min</td>
<td>-37.00</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
<tr>
<td>G-max</td>
<td>-8.57</td>
<td>1.000</td>
<td>AOVRV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of parameters</th>
<th>AOVRV</th>
<th>OVRV</th>
<th>IDM</th>
<th>AFVD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Proposed asymmetric model outperforms baseline models

• AOVRV outperforms other models significantly (e.g., IDM, AFVD)

• Asymmetric model is able to recreate asymmetric car following behavior
  – Accurately matches both acceleration and braking

• String stability is analyzed via linearized hybrid system string stability: all string unstable
String stability criterion of the AOVRV model: a linearized approach

- Assume N vehicles on a ring road L, when at steady state:
  \[ \dot{x}_e = \frac{s_e - \eta}{\tau} \quad s_e = \frac{L}{N} \quad x_n^{(0)}(t) = s_en + \dot{x}_e t \]

- Assume a small perturbation \( y_n \):
  \[ x_n = x_n^{(0)} + y_n, \quad |y_n| \ll 1 \quad \dot{h}_n = \dot{y}_{n-1} - \dot{y}_n \]

- Linearize the perturbation at the equilibrium point:
  \[ \ddot{y}_n = k_{1D} \tau \left( g_D \dot{h}_n - \dot{y}_n \right) \Theta(-\dot{h}_n) + k_{2D} \Theta(-\dot{h}_n) \dot{h}_n + k_{1A} \tau \left( g_A \dot{h}_n - \dot{y}_n \right) \Theta(\dot{h}_n) + k_{2A} \Theta(\dot{h}_n) \dot{h}_n \]

- Solve \( y(n, t) \) with Fourier series for string stability criterion

[Bando 1996.]
String stability analysis:

- String stable criterion:

\[
g = \begin{cases} 
g_D = V_D'(s_e) = \frac{1}{\tau} < \frac{k_1 p}{2} + k_2 \dot{s} < 0 \\
g_A = V_A'(s_e) = \frac{1}{\tau} < \frac{k_1 q}{2} + k_2 \dot{s} < 0
\end{cases}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Motion</th>
<th>(k_1)</th>
<th>(k_2)</th>
<th>String stability</th>
<th>Veh. string stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Accel.</td>
<td>0.015</td>
<td>0.091</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>Decel.</td>
<td>0.029</td>
<td>0.196</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td>II</td>
<td>Accel.</td>
<td>0.900</td>
<td>0.900</td>
<td>Stable</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>Decel.</td>
<td>0.029</td>
<td>0.196</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td>III</td>
<td>Accel.</td>
<td>0.015</td>
<td>0.091</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>Decel.</td>
<td>0.900</td>
<td>0.900</td>
<td>Stable</td>
<td>Unstable</td>
</tr>
<tr>
<td>IV</td>
<td>Accel.</td>
<td>0.900</td>
<td>0.900</td>
<td>Stable</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>Decel.</td>
<td>0.900</td>
<td>0.900</td>
<td>Stable</td>
<td>Stable</td>
</tr>
</tbody>
</table>

\[\tau = 2.168\] and \[\eta = 12.967\]
Task 1.2 A continuous asymmetric car following model considering vehicle actuation delay (In progress)

Recall: \[ \ddot{x}(t) = k_1D\tau (V_D(s) - \dot{x}) \Theta(-\dot{s}) + k_{2D}\Theta(-\dot{s})\dot{s} + k_{1A}\tau (V_A(s) - \dot{x}) \Theta(\dot{s}) + k_{2A}\Theta(\dot{s})\dot{s} \]

\[ V_A(s) = V_D(s) = \frac{s - \eta}{\tau} \]

\[ \Theta(\dot{s}) = \begin{cases} 
0 & \text{if } \dot{s} \leq 0 \\
1 & \text{if } \dot{s} > 0 
\end{cases} \]

Challenge: Approximate discontinuity by substituting a smooth term

Goal:

- Ensuring smooth and realistic acceleration without drastic changes
- Reducing simulated velocity errors
Modeling ACC with actuation delay

- Assume a delay between a change in speed and spacing and the response of the following vehicle

\[ \ddot{x}_j(t) = f(s_j(t - t_d), \dot{x}_j(t), \dot{x}_{j-1}(t - t_d)) \]

- Use the proposed ODE model as a baseline framework

- Calibrate with Delay Differential Equation
Planning on future high-level automated vehicles

- **Level 0**: No automation
- **Level 1**: Driver assistance
- **Level 2**: Partial automation
- **Level 3**: Conditional automation
- **Level 4**: High automation
- **Level 5**: Full automation

- Predict fully automated vehicle dynamics based on partially automated vehicle dynamics
- Investigate the impacts of mixed traffic at different levels of automation
Completed work:
• Proposing asymmetric car following model to account for asymmetric ACC car following behavior
• Calibrating and simulating with the proposed model for future utilization in impact studies.
• Modeling stochastic human driven behavior with an accurate dataset
• Task 1.1 is published on IEEE Transactions on Intelligent Transportation Systems, 2022

Planned work:
• Improve the proposed asymmetric car following model for higher accuracy
• Incorporate ACC dynamics with an actuation delay
• Predict future AV dynamics with higher level of automation
Task 2: Impacts of mixed autonomy traffic flow (Completed)

- Task 2.1: Impacts of mixed autonomy traffic flow on highway string stability (Completed)

- Task 2.2: Impacts of mixed autonomy traffic flow on highway throughput and fundamental diagram (Completed)

- Task 2.3: Impacts of mixed autonomy traffic flow on vehicle fuel consumption and emissions (Completed)
Commercially available ACC vehicles may result in more traffic oscillations.
Simulation on a ring road at different MPRs (60% - 100%)

[Simulation animation by Bhadani, R.]
Traffic string stability: linearized vehicle dynamics

For a generic car following model $\ddot{x}_j(t) = f(s_j(t), \dot{s}_j(t), \dot{x}_j(t))$ at equilibrium $f(s_j^*, 0, \dot{x}_j^*) = 0$

Stability depends on the growth rate of a perturbation:

$$\lambda_2 := \frac{f_s}{f \dot{x}} \left( \frac{f_x^2}{2} - f_s f \dot{x} - f_s \right)$$

If $\lambda_2 < 0$ the car following model is string stable

If $\lambda_2 > 0$ the car following model is string unstable

Rational driving constraints (RDC)

- $\frac{\partial f}{\partial s} := f_s \geq 0$
- $\frac{\partial f}{\partial \dot{s}} := f_s \geq 0$
- $\frac{\partial f}{\partial \dot{x}} := f_s \leq 0$

More space: speed up
Large speed difference: speed up
Higher speeds: less acceleration

[Wilson and Ward, 2010]
Homogeneous traffic string stability - IDM

Recall: IDM:
\[ \ddot{x} = a \left( 1 - \left( \frac{\dot{x}}{v_0} \right)^\delta - \left( \frac{\hat{s}(\dot{x}, \hat{s})}{s} \right)^2 \right) \]
\[ \hat{s}(\dot{x}, \hat{s}) = \eta + \tau \dot{x} - \frac{\dot{x} \hat{s}}{2\sqrt{ab}} \]

Fully automated AV \( \lambda_2 \) vs. \( v_e \)

Commercially-available ACC \( \lambda_2 \) vs. \( v_e \)

Note: Below zero (\(<0\)) indicates stable case
Heterogeneous traffic string stability – ACC mixed with HV

Heterogeneous traffic is **string stable** when:

\[
\sum_n \left[ \frac{(f_n^{\bar{x}})^2}{2} - f_s^n f_x^n - f_s^n \right] \left[ \prod_{m \neq n} f_s^m \right]^2 \geq 0
\]

where \( n \) and \( m \) represent different types of traffic

When two types of traffic (ACC and Human) mixed **string stable**:

\[
S = \eta (f_s^{HV})^2 \left[ \frac{(f_x^{ACC})^2}{2} - f_s^{ACC} f_x^{ACC} - f_s^{ACC} \right] + (1-\eta) (f_s^{ACC})^2 \left[ \frac{(f_x^{HV})^2}{2} - f_s^{HV} f_x^{HV} - f_s^{HV} \right] \geq 0
\]

where \( \eta \) is the market penetration rate (MPR) of ACC (%)

- We consider 20\%, 40\%, 60\%, 80\% MPRs in the mixed traffic flow study.

[Ward 2009]
More penetration rates of commercially-available ACC, lower string stabilities

Stability vs. $v_e$ of human-driven vehicles mixed with 20%, 40%, 60%, 80%

Fully automated AV

Commericaly-available ACC (ACC-A-max)

Note: Above zero (>0) indicates stable case
Initial guess: fuel consumption and emission model VT-Micro

\[ \ln(MOE_e) = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} (K_{\alpha,\beta}^e \times v^\alpha \times \ddot{x}^\beta) \]

- MOE is the instantaneous fuel consumption (L/s) or emissions like CO\(_2\), HC, NOx (Gram/s)
- \(K_{\alpha,\beta}^e\) is a regression coefficient for a specific MOE at speed power \(\alpha\) and acceleration power \(\beta\)
- \(v\) is the instantaneous speed (m/s)
- \(\ddot{x}\) is the instantaneous acceleration (m/s\(^2\))

Use the trajectories from simulations to evaluate emissions

[Ahn, et al, 2002]
Commercially available ACC vehicles may result in more traffic oscillations.
Commercially available ACC vehicles may increase fuel consumption and emissions

- Fully automated vehicles may slightly decrease fuel consumption or emissions
- Commercially available ACC vehicles generally increase the fuel consumption
- Up to a 76% increase in fuel consumption, and 97% increase in NOx as a result of commercially available ACC vehicles
Completed work:

- Increased vehicle autonomy will change traffic flow dynamics at the macroscopic level
  - Long term trend: with full automation
    - increased capacity
    - decreased fuel consumption and emissions
    - increased string stability
  - Short term trend: with partial automation
    - decreased capacity
    - increased fuel consumption and emissions
    - decreased string stability

- Task 2.1, 2.2 are published on *Transportation Research Part C: Emerging Technologies*, 2021
- Task 2.3 is submitted to *Journal of Transportation Engineering, Part A: Systems*, 2022
Task 3: Traffic control in the presence of mixed autonomy traffic (Planned)

- Task 3.1: Macroscopic on-ramp traffic flow dynamics for mixed autonomy traffic (In progress, Summer 22)

- Task 3.2: Extending ramp metering control to mixed autonomy traffic flow with varying degrees of automation (Planned, Fall 22)

- Task 3.3: Development of advanced ramp metering strategies (Planned, Spring 23)
Task 3.3: Development of advanced ramp metering strategies

- Advanced ramp metering strategies may improve accuracy in regulating traffic.

- Ramp metering strategies are subjected to change due to traffic flow at different levels of automation.

- E.g., An asymptotic stable controller for mixed autonomy traffic via sliding surface control.

- Challenge:
  1) Determine the sliding surface in the presence of mixed autonomy.
  2) Design of sliding surface controller and analyze stability.

[Rajamani, ME 8282 class; Mammar, 2006]
The end: modeling and control mixed autonomy traffic

• Task 1: Accurately modeling vehicle dynamics
  – Modeling asymmetric ACC driving behavior ✓
  – Modeling human driver stochastic driving behavior ✓
  – Improvement of modeling ACC vehicles and planning on higher-level automated vehicles (Summer 22)

• Task 2: Assess the impacts of mixed autonomy traffic flow
  – String stability ✓
  – Highway throughput and fundamental diagram ✓
  – Fuel consumption and emissions ✓

• Task 3: Traffic control in the presence of mixed autonomy traffic
  – Macroscopic on-ramp traffic flow dynamics for mixed autonomy traffic (Summer 22)
  – Extending ramp metering control to mixed autonomy traffic flow (Fall 22)
  – Development of advanced ramp metering strategies (Spring 23)
Selected publications

Journal publications


Conference proceedings

Task 1.3*: Modeling human driver stochastic driving behavior

- Optimal velocity-follow the leader (OV-FTL) model (Cui, et al 2017)
  \[
  \ddot{x} = a(V(s) - \dot{x}) + b \frac{\dot{s}}{s^\nu} \\
  V(s) = V_m \frac{\tanh(s/d_0 - 2) + \tanh(2)}{1 + \tanh(2)}
  \]

- Optimal velocity relative velocity model (OVRV)
  \[
  \ddot{x}(t) = k_1 [s - (\eta + \tau \dot{x})] + k_2 \dot{s}
  \]

  \[
  \ddot{x} = a \left(1 - \left(\frac{\dot{x}}{v_0}\right)^\delta - \left(\frac{\dot{s}(\dot{x}, \dot{s})}{s}\right)^2\right) \\
  \dot{s}(\dot{x}, \dot{s}) = \eta + \tau \dot{x} - \frac{\dot{x} \dot{s}}{2\sqrt{ab}} \\
  \]
  \(a, b\): maximum acceleration and comfortable braking rate \(v_0\): desired limit speed \(s_0\): jam distance (minimum distance) \(\tau\): time-gap (headway) \(\delta\): acceleration exponent
A dataset to investigate human-driven unstable behavior

<table>
<thead>
<tr>
<th>Next Generation SIMulation (NGSIM, CA)</th>
<th>HighD data (Germany)</th>
<th>Arizona Ring Experiment Data (ARED)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Relatively little data</td>
<td>• 400 meters roadway</td>
<td>• 8 Experiments: 20-22 vehicles</td>
</tr>
<tr>
<td>• Inaccurate in capture of vehicles</td>
<td>• Effects caused by bottlenecks or geometric factors</td>
<td>• Duration around 400 secs</td>
</tr>
<tr>
<td>• Effects caused by bottlenecks or geometric factors</td>
<td></td>
<td>• Comprehensive to consider the exclusive traffic oscillations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Accurate in capture stop and go waves</td>
</tr>
</tbody>
</table>

Calibration and simulation vehicle trajectories: a similar approach with modeling ACC, however...

Simulation process:

\[
\begin{bmatrix}
    x \\
    \dot{x}
\end{bmatrix}_{t+\Delta t} = \begin{bmatrix}
    x \\
    \dot{x}
\end{bmatrix}_t + \begin{bmatrix}
    \dot{x} \\
    f(\theta, s, \dot{x}, \ddot{s})
\end{bmatrix}_t \Delta t + \begin{bmatrix}
    \epsilon \\
    0
\end{bmatrix}_t
\]

Gaussian noise:

\[\epsilon \sim \mathcal{N}(\mu, \sigma)\]

- Task 1.3* is published in *Forum on Integrated and Sustainable Transportation Systems, 2020*
Traffic flow characteristic metrics: Results

<table>
<thead>
<tr>
<th></th>
<th>Exp A (testing)</th>
<th>OV-FTL $\epsilon \sim N(0, 0.05)$</th>
<th>OVRV $\epsilon \sim N(0, 0.05)$</th>
<th>IDM $\epsilon \sim N(0, 0.05)$</th>
<th>IDM $\epsilon \sim N(0, 0.4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\delta}_v\overline{\nu}_n$</td>
<td>0.95</td>
<td>0.57</td>
<td>0.61</td>
<td>0.31</td>
<td>0.83</td>
</tr>
<tr>
<td>$\bar{\delta}_s\overline{n}_s$</td>
<td>2.19</td>
<td>1.81</td>
<td>1.40</td>
<td>0.64</td>
<td>2.05</td>
</tr>
<tr>
<td>$\bar{\delta}_v\overline{t}_t$</td>
<td>0.71</td>
<td>0.42</td>
<td>0.40</td>
<td>0.25</td>
<td>0.77</td>
</tr>
<tr>
<td>$\bar{\delta}_s\overline{t}_t$</td>
<td>3.02</td>
<td>4.19</td>
<td>3.71</td>
<td>2.58</td>
<td>3.54</td>
</tr>
<tr>
<td>$t_s$</td>
<td>10.10</td>
<td>8.00</td>
<td>7.03</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Explanation of $\lambda_2$: growth rate of waves

$$\lambda^2 + \{ f_{\Delta v}(1 - e^{-i\theta}) - f_v \} \lambda + f_s(1 - e^{-i\theta}) = 0.$$  

$$\lambda_+(\theta) = i\lambda_1 \theta + \lambda_2 \theta^2 + i\lambda_3 \theta^3 + \lambda_4 \theta^4 + \ldots,$$

Figure 4: Plots of the growth rate $\lambda_r$ as a function of $\theta$: (a) string-stable; (b) string-unstable. The transition to instability is given by a change in the sign of the second derivative $\lambda_2$ at $\theta = 0$. 